I. Solve the initial value problems.
1. \( y'' + 4y = 3 \sin 2t, \ y(0) = 2, \ y'(0) = 0. \)
2. \( y'' - 2y' + y = te^t + 4, \ y(0) = 1, \ y'(0) = 0. \)

II. Suppose \( y'(t) = F(y(t)) \), where the graph of \( F(y) \) is given below. Carefully draw several solution curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume \( y(t) \) and \( t \) are non-negative.

III. These are similar to problems 1-6 in 10.5.
1. Consider the PDE, \( p(x)u_{xx} + u_t = 0. \) Suppose \( u(x,t) = X(x)T(t). \) Show that for some constant \( \sigma \) it follows that

\[
p(x)X'' + \sigma X = 0 \quad T' - \sigma T = 0.
\]

2. Consider the PDE, \( u_{xx} + u_{yy} = u_t. \) Suppose \( u(x,y,t) = X(x)Y(y)T(t). \) Show that for some constants \( \alpha \) and \( \sigma \) it follows that

\[
T' + \sigma T = 0 \quad X'' + \alpha X = 0 \quad Y'' + (\sigma + \alpha)Y = 0.
\]
IV. Now for the fun stuff.

1. Consider a vibrating ideal string (that is assume the wave equation is valid) with $a = 1$, $L = 3$, with initial displacement $f(x)$ given below.

![Graph of initial displacement](image1)

Find $u(x,t)$ and plot it for $t = 0.0$, $t = 0.001$, $t = 0.01$, $t = 0.1$, and $t = 0.5$.

2. Consider a vibrating ideal string (that is assume the wave equation is valid) with $a = 1$, $L = \pi$, with initial displacement $f(x) = \sin 5x$. Find $u(x,t)$ and plot it for $t = 0.0$, $t = 0.001$, $t = 0.01$, $t = 0.1$, and $t = 0.5$.

3. Consider a metal rod of length 2 and $\alpha = 1$ with the initial temperature shown below.

![Graph of initial temperature](image2)

Find $u(x,t)$ and plot it for $t = 0.0$, $t = 0.001$, $t = 0.01$, $t = 0.1$, and $t = 0.5$.

4. Animate any of these for extra credit. You can just show me the animation on your laptop/tablet or email it to me.