

Due Monday, February 6

I. Find the general solution to each of the following differential equations. If an initial condition is given, also find the particular solution.

1. $y' = y^3 \sin x$.
2. $xy' + 2y = \sin x$, assume $x > 0$.
3. $xy^2y' + y^3 = 1$, assume $x > 0$.
4. $ty' + y = e^t$, assume $t > 0$.
5. $y' = e^x \cos^2 y$.
6. $y' + \frac{y}{t-1} = \frac{1}{t-3}$, assume $1 < t < 3$.
7. $x^2y' - 3y = 2$, assume $x > 0$.
8. $y' = \frac{y^2 + 2xy}{x^2}$.
9. $y' = xy + 2y + 3x + 6$.
10. $x^2y' + 2xy = y^3$, assume $x > 0$.
11. $y' = xy$, $y(1) = 1$.
12. $y' = \frac{x^2 + y^2}{xy}$.
13. $z' - 3z = e^x$, with $z(0) = 2$.
14. $y' + \frac{y}{2} = \sin \frac{x}{2}$, with $y(\pi) = 6$.

II. Use a computer to plot the direction field for each differential equation below. Use $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$. Also plot two solution curves.

1. $y' = x - 3y$.
2. $y' = x - y^2$.
3. $y' = \sin(\pi xy)$.

III. Do the following problems. Show all steps. Explain what you are doing.

1. A storage tank contains 2000 gal of gasoline that initially has 100 lb of an additive dissolved in it. At $t = 0$ gasoline containing 2 lb of the additive per gallon is pumped into the tank at 40 gal/min. The gasoline in the tank is kept well mixed. It is drained out at 40 gal/min. How many pounds of the additive are dissolved in the remaining gasoline after 30 minutes?
2. A chemical spill has polluted a pond. Your company, Clean Ponds Inc., has been contracted to clean the pond. Federal regulations require that 90% of the pollutant be removed within one month (31 days). The pond has 600,000 gallons of water in it.

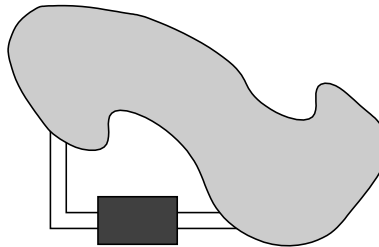
The following pump/filter systems are available:

| name | Econo-Pump and Filter | Super Filter System | Pump Master with Filter |
|------------|-----------------------|---------------------|-------------------------|
| cost | \$150,000 | \$250,000 | \$350,000 |
| pump rate | 4000 gal/hr | 3000 gal/hr | 5000 gal/hr |
| efficiency | 40 % | 75 % | 65 % |

Notes to Table: The cost listed is the minimum rental charge per month. Filter efficiency is the percentage of pollutant removed on each pass through the filter.

A) Which pump/filter system should you get?

Note: Since the pump sends the water back into the pond after it is filtered, the water coming in is part new water and part filtered water. Assume the water in the pond is well mixed. (See figure.)



B) Because of a lawsuit by an environmental group you must get 98% of the pollutant out in one month (31 days). Now what do you do?

3. You have a motor boat. Assume that its velocity is modeled by

$$v' = F_m - kv,$$

where F_m is the force of the motor in pounds, k is a constant, and v is velocity in feet/second. As an experiment you run your boat with various motor forcings and measure the terminal velocity. The data is in the table below.

| F_m | v | k |
|-------|-----------|-----|
| 2 | 4.641589 | |
| 4 | 7.368063 | |
| 6 | 9.654894 | |
| 8 | 11.690710 | |
| 10 | 13.572088 | |

For each experimental run compute the value of k . Is the assumed model a good one? Explain. If it is not, find a better one.