Homework Set 4  
Due Monday, February 14

I. Find the solution to each of the following. You may leave your answer in the form of a relation.

1.  \( \frac{y'}{y} = -\frac{2x + y}{x + 2y} \), with \( y(1) = 1 \).

2.  \( 3x^2 + y^2 + 2xyy' = 0 \), with \( y(1) = 2 \).

3.  \( (2y - x)y' = y - 4x \), with \( y(2) = 1 \).

4.  \( \cos x \sec y + \sin x \sin^2 y \frac{dy}{dx} = 0 \), with \( y(\pi/6) = \pi/4 \).

5.  \( 2y^2 - 6xy + (3xy - 4x^2)y' = 0 \), with \( y(1) = 1 \). Hint: It is not exact, but will become exact if you multiply through by \( xy \).

II.

1. For #1 above, plot the solution curve with a computer.

III. Find the particular solution to each of the following.

1.  \( y'y'' = 4t \), with \( y(1) = 5 \) and \( y'(1) = 2 \).

2.  \( \frac{d^2y}{dt^2} = \frac{3}{2}y^2 \), with \( y(0) = 1 \) and \( y'(0) = 1 \).

IV.

1. At 5 a.m. you put a 60° beer in a 40° refrigerator. You get up at 8 a.m. expecting to have a cold one with your Honey Puffs cereal but to your horror you find that someone has taken your beer and left it on the table. The beer is now 65°. (Room temperature, you note, is 70°.)

   You know roommate #1 left for work at 6 a.m. and so go to confront roommate #2. But, he swears that he has been asleep the whole time. You bet him a six pack you can prove it was he who left the beer out. He folds his arms, smiles and says “you’re on!”

   Recall Newton’s Law of Cooling:  \( \frac{dT}{dt} = k(T_a - T(t)) \).

   You put the beer back in the frig and wait 30 minutes (while roommate #2 watches cartoons thinking about his six pack). When you take the beer out it has cooled to 50°. You calculate \( k \). Now, win the bet!

2. The number of algae cells in a tank of water grows according to

\[
A'(t) = \begin{cases} 
0.2 \left( 1 - \frac{A(t)}{100} \right) A(t) & \text{light on, and} \\
-0.2A(t) & \text{light off.}
\end{cases}
\]
In words, the carrying capacity drops from 100 (billion cells) to zero when
the light goes out. At $t = 0$ you start a ten (10) hour experimental run
with $A(0) = 25$ and plan to keep the light on. When you come back at
$t = 10$ you discover that the light bulb has burned out. You measure
$A(10)$ to be 15. What time did the light bulb burn out?

3. Suppose $y'(t) = F(y(t))$, where the graph of $F(y)$ is given below. Carefully
draw the integral curves for this equation. What are the equilibrium
solutions? What are their stability types? Describe the initial concavity
of the solution curves. Assume $y(t)$ and $t$ are nonnegative.