Due Monday February 20

I. For each differential equation below, find the general solution, the particular solution for the initial values given, and graph it. (You can use a computer or calculator, but label the intercepts and roughly the value of any extrema.)

1. \( y'' - y' - 2y = 0, \ y(0) = 1, \ y'(0) = 2. \)
2. \( 4y'' - 4y' + y = 0, \ y(1) = 2, \ y'(1) = -1. \)
3. \( y'' - 4y' + 13y = 0, \ y(0) = 4, \ y'(0) = -1. \)
4. \( y'' + 9y = 0, \ y(0) = 2, \ y'(0) = 1. \)

II. These involve equations with nonconstant coefficients.
1. Consider \( t^2y'' + 4ty' - 10y = 0. \) Suppose that \( y(t) = t^r \) is a solution. What are the allowed values of \( r \)? What do you think the general solution is? Plug it in and check. Find the particular solution if \( y(1) = 2 \) and \( y'(1) = 3. \) On what interval do you think it will be valid?

2. Consider \( y'' - \frac{1}{x}y' + 4x^2y = 0. \) Assume \( x > 0. \) (a) Show that \( y_1(x) = \sin(x^2) \) is a solution. (b) Let \( y_2(x) = v(x) \sin(x^2). \) Use the reduction of order method to find another solution that is not a multiple of \( y_1. \)

3. Consider \( t^2y'' - ty' + y = 0. \) Suppose \( t^r \) is a solution. Find \( r. \) Use the Reduction of Order Method, as in #2, to find a second solution. What is the general solution?

III. [Extra Credit]. Consider \( y' = t(5 - y), \ y(0) = 0. \) Find the solution. Use a spreadsheet program to compare the solution with the Euler Method and Improved Euler Method approximations for stepsize \( h = 0.1 \) for \( t_n = nh, \) from 0.0 to 5.0.

See the spreadsheet link next to this hwk link for an example where the equation was \( y' = t - 2y. \)