I. For each differential equation below, find the general solution.

1. \( 9y'' + 6y' + y = x^2 \)
2. \( 9y'' + 6y' + y = e^{-\frac{x}{3}} \)
3. \( y'' + 3y' - 4 = 2x + e^x \)
4. \( y'' + 3y' - 4 = \sin x \)
5. \( y'' + 4y = x \)
6. \( y'' + 4y = \cos x \)
7. \( y'' + 4y = \cos 2x \)

II. Determine if each pair of functions is linearly dependent or independent on the given interval.

1. \( \{ x^2 + x, x \} \) on \(( -\infty, \infty)\)
2. \( \{ \sin x, \sin 2x \} \) on \(( -\infty, \infty)\)
3. \( \{ \ln x, \ln \frac{1}{x} \} \) on \(( 0, \infty)\)
4. \( \{ e^x, e^{x+1} \} \) on \(( -\infty, \infty)\)
5. \( \{ \frac{x+1}{x}, \frac{x}{x-1} \} \) on \(( -1, 1)\)

III. For a set of three functions, \( \{ f_1, f_2, f_3 \} \), each twice differentiable, the Wronskian is defined to be

\[
\begin{vmatrix}
 f_1 & f_2 & f_3 \\
 f_1' & f_2' & f_3' \\
 f_1'' & f_2'' & f_3'' 
\end{vmatrix}
\]

A set of three functions defined on an interval \( I \) is linearly dependent if there exist real numbers, \( C_1, C_2 \) and \( C_3 \), not all zero such that

\[
C_1 f_1(x) + C_2 f_2(x) + C_3 f_3(x) = 0 \quad (*)
\]

for all \( x \in I \). Otherwise a set is linearly independent. It is known that if a set of three twice differentiable functions is linearly dependent then the Wronskian is zero on \( I \). (Extra Credit: Prove this!)

Find the Wronskian of the four sets below. If it is not always zero conclude the set is linearly independent. If it is always zero find values for \( C_1, C_2 \) and \( C_3 \), not all zero, that satisfy \((*)\). The interval for each is the whole real line.

1. \( \{ \sin^2 x, \cos^2 x, \cos 2x \} \)
2. \( \{ \sin x, \sin 2x, \sin 3x \} \)
3. \( \{ x^2 + 3x, x + 2, x^2 + x - 4 \} \)
4. \( \{ x^2, x^2 + x, x^2 + x + 1 \} \)

IV. We consider differential equations of the form \( y'' + p(t)y' + q(t)y = 0 \).

Assume \( p(t) \) and \( q(t) \) are continuous on the whole real line.

1. Explain why both \( y_1 = t^3 \) and \( y_2 = t^4 \) cannot be solutions.
2. Explain why both \( y_1 = \sin t \) and \( y_2 = t^2 + t \) cannot be solutions.

V. 1. Find a differential equation of the given form \( y'' + p(x)y' + q(x)y = 0 \) that has \( \{ x+1, x+2 \} \) as a fundamental solution set. On what interval is it valid?

2. Find a differential equation of the given form \( y'' + p(x)y' + q(x)y = 0 \) that has \( \{ \sin x, e^x \} \) as a fundamental solution set. On what interval is it valid?