I. For each differential equation below, find the general solution.

1. \( y'' + y = \csc t, \quad 0 < t < \pi. \)
2. \( y'' + y = \sec t \tan t, \quad -\pi/2 < t < \pi/2. \)
3. \( \frac{d^2y}{dx^2} - \frac{dy}{dx} = x^3 \)
4. \( y'' + 4y' + 5y = 10. \)
5. \( x^2y'' + xy' - y = x. \)
6. \( y' = \frac{-3x^2+y}{x+2y} \)
7. \( 4y'' + 4y' + y = e^{-t}/2 \)
8. \( y' + ty = ty^3 \)

II. 1. A 9 lb object is attached to the lower end of a spring whose upper end is attached to the ceiling. The spring constant \( k \) is 1 lb/ft. The resistance to the motion is \( \gamma = 0.25 \text{ lb-s/ft} \) times the velocity of the object. The object is set in motion by pulling it down 1 ft and then letting it go.

Set up a differential equation to model this mass-spring system. Solve it and plot the solution. (Don’t forget to convert weight to mass.)

2. A 9 lb object is attached to the lower end of a spring whose upper end is attached to the ceiling. The spring constant \( k \) is 1 lb/ft. The resistance to the motion is \( \gamma = 0.25 \text{ lb-s/ft} \) times the velocity of the object. The system is in equilibrium (no motion). Then, at time zero, a 1 lb hawk lands gently onto the object. Now what happens?

Set up a differential equation to model this mass-spring system. Solve it and plot the solution.

3. A 9 lb iron object is attached to the lower end of a spring whose upper end is attached to the ceiling. The spring constant \( k \) is 1 lb/ft. The resistance to the motion is \( \gamma = 0.25 \text{ lb-s/ft} \) times the velocity of the weight. The system is in equilibrium (no motion). Then, at time zero, a oscillating magnet on the floor is turned on. It exerts a force on the object of \( 2 \sin t \) pounds - positive is down and negative is up.

Set up a differential equation to model this mass-spring system with external forcing. Solve it and plot the solution.

III. Essay question: Look up non-linear springs. Describe several types and what they might be used for. List references you use. Where would I buy one?