I. For each differential equation below, find the general solution.

1. \( xy' + 2y + 3x = 0 \)
2. \( 4y'' + 17y' + 4y = \sin t \)
3. \( y'' + 9y = 9 \sec^2(3t), -\pi/6 < t < \pi/6 \)
4. \((e^x + 1)y' = y(1 - e^x)\)
5. \( \frac{d^2H}{d\theta^2} + \frac{dH}{d\theta} - 2H = \theta e^\theta \)
6. \( w'' + 9w = 9t - \cos 3t \)
7. \( 3y''' - 2y'' + 15y' - 10y = 0 \)
8. \( y'''' - y = 0 \)
9. \( y''' - y'' - 2y' = t^2 + t \)

II.

1. Show that \( y_1(t) = \tan t \) is a solution to \( y'' - (2 \sec^2 t)y = 0 \). Find a second solution \( y_2(t) \) that is linearly independent from \( y_1(t) \).
2. Let \( g(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } 0 \leq t < 5 \\ 0 & \text{for } t \geq 5 \end{cases} \)

Let \( h(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } 0 \leq t < 4 \\ 0 & \text{for } t \geq 4 \end{cases} \)

(a) Find a differentiable solution to \( y'' + \pi^2 y = g(t), y(0) = y'(0) = 0 \), then graph it over \( t \in [0, 10] \)

(b) Find a differentiable solution to \( y'' + \pi^2 y = h(t), y(0) = y'(0) = 0 \), then graph it over \( t \in [0, 10] \)

3. Find the general solution to \( y'' + 3y' + 2y = \cos^2 t \). Although this is not covered by the method of undetermined coefficients given in your textbook, try \( y_p = A \cos^2 t + B \sin t \cos t + C \sin^2 t \), and see what happens!

4. Let \( \epsilon(n) = \begin{cases} 1 & \text{for } n = 0, 1, 4, 5, 8, 9, ... \\ -1 & \text{for } n = 2, 3, 6, 7, 10, 11, ... \end{cases} \)

Let \( f(x) = \sum_{n=0}^{\infty} \epsilon(n) \frac{x^n}{n!} \).

Show by direct substitution that \( y = f(x) \) solves the initial value problem \( y'' + y = 0, y(0) = y'(0) = 1 \).