

Name: _____

ONLY SCIENTIFIC CALCULATORS ALLOWED

1. [10 points] Find the general solution to
- $y'' - y' - 6y = 2\sin t$
- .
- $+ 0 \cos t$

$$r^2 - r - 6 = 0$$

$$(r-3)(r+2) = 0$$

$$r = 3, -2$$

$$y_h = C_1 e^{-2t} + C_2 e^{3t}$$

$$y_p = A \cos t + B \sin t$$

$$y_p' = -A \sin t + B \cos t$$

$$y_p'' = -A \cos t - B \sin t$$

$$y_p'' - y_p' - 6y_p = (-A - B - 6A) \cos t + (-B - A - 6B) \sin t$$

$$-7A + B = 0$$

$$B = -7A$$

$$-A - 7B = 2$$

$$-A + 49A = 2$$

$$48A = 2$$

$$A = \frac{2}{48} = \frac{1}{24}$$

$$B = -\frac{7}{24}$$

$$y = y_h + y_p = C_1 e^{-2t} + C_2 e^{3t} + \frac{1}{24} \cos t - \frac{7}{24} \sin t$$

2. [10 points] Find the general solution to
- $y'' - y' - 6y = 3e^{2t}$
- .

$$\text{Again } y_h = C_1 e^{-2t} + C_2 e^{3t}$$

$$y_p = A e^{2t}$$

$$y_p' = 2A e^{2t}$$

$$y_p'' = 4A e^{2t}$$

$$y_p'' - y_p' - 6y_p = (4A - 2A - 6A) e^{2t}$$

$$-4A = 3$$

$$A = -\frac{3}{4}$$

$$y = y_h + y_p = C_1 e^{-2t} + C_2 e^{3t} - \frac{3}{4} e^{2t}$$

3. [10 points] Find the general solution to
- $y'' + y = t^2$
- .

$$r^2 + 1 = 0$$

$$r = \pm i$$

$$y_h = C_1 \cos t + C_2 \sin t$$

$$y_p = At^2 + Bt + C$$

$$y_p' = 2At + B$$

$$y_p'' = 2A$$

$$y'' + y = 2A + At^2 + Bt + C = t^2$$

$$\text{Thus } A=1, B=0 \text{ and } 2A+C=C$$

$$\text{Hence } C=-2$$

$$y_p = t^2 - 2$$

$$y = y_h + y_p = C_1 \cos t + C_2 \sin t + t^2 - 2$$