Math 305 Homework Set 1 Spring 2018

Due Monday, January 22

I. Draw the following graphs without the aid of a calculator or computer. Label zeros and the y-intercept.

1. $y = 3e^{-2x}$
2. $y = e^{-x}\cos x$
3. $y = \dfrac{1}{1+e^x}$
4. $y = \dfrac{1}{2}\sin(3x+\pi)$ (two cycles)
5. $y = e^{2\ln|x|}$
6. $y = x\sin x$

II. Do the following integrals. Take the derivative to check your answers.

1. $\displaystyle\int \frac{1}{x+1}\,dx$
2. $\displaystyle\int \frac{1}{\sqrt{x+1}}\,dx$
3. $\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,dx$
4. $\displaystyle\int \frac{1}{1+x^2}\,dx$
5. $\displaystyle\int \frac{x}{1+x^2}\,dx$
6. $\displaystyle\int \frac{x}{1+x}\,dx$
7. $\displaystyle\int \frac{t^2}{1+t^2}\,dt$
8. $\displaystyle\int \sec^2 x\,dx$
9. $\displaystyle\int \sec x\,dx$
10. $\displaystyle\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$
11. $\displaystyle\int x^2e^x\,dx$
12. $\displaystyle\int_0^4 \frac{5}{3x+1}\,dx$
13. $\displaystyle\int \frac{1}{x+x^2}\,dx$
14. $\displaystyle\int \frac{1}{1-x^2}\,dx$
15. $\displaystyle\int x\sin x\,dx$
16. $\displaystyle\int \sin^3 x\cos^2 x\,dx$
17. $\displaystyle\int \cot x\,dx$
18. $\displaystyle\int \tan^2 x\,dx$

III. Find all functions $y(x)$ that satisfy the following conditions.

1. $y' = 5x/y$
2. $y' = \dfrac{\sqrt{x}}{2y}$
3. $y' = 3y$
4. $y' = x(1+y)$
5. $y' = \dfrac{3}{4}\sqrt{x}$ and $y(0) = 10$
6. $y' = 3y$ and $y(0) = 10$
7. $y'' = g$, $y(0) = 4$ and $y(10) = 12$ (g is a constant)
8. $y'' = \sin x$, $y(0) = 1$ and $y'(0) = 7$

IV. Answer the following.

1. If radium has a half-life of 1620 years, what percentage of a sample will be left after 500 years?

2. If a sample of radioactive material decays at a rate of 10% per day, what is its half-life?

V. Read about *Taylor series* if you need to then do the following.

1. Find the Taylor series of $y = e^{x^2}$ centered about $x = 0$. What is the radius of convergence?

2. Find the Taylor series of $y = 1/x$ centered about $x = 1$. What is the radius of convergence?

Math 305 Homework Set 2 Spring 2018

Due Monday January 29

I. Find the general solution to each of the separable differential equations below. If an initial condition is present also find the particular solution. These problems are based on Section 2.2.

(1) $y' = y$ and $y(0) = -6$.
(2) $y' = y$ and $y(1) = e^2$.
(3) $y' = \cos(2x) + 2$ and $y(\pi) = 8$.
(4) $y' = y^2 e^x$ and $y(0) = 2$.
(5) $y' = e^{x+y}$.
(6) $y' = x^2 \cos^2(y)$
(7) $y' = y^2 e^x$
(8) $y' = x \sin(x) \cot(y)$ and $y(\pi/4) = 6$.
(9) $y' = x^3 y^2 + xy^2 + 2y^2 + x^3 + x + 2$ and $y(0) = 1$.
(10) $y' = xy^3 \sqrt{1 + x^2}$ and $y(0) = 1$.

II. Describe the set of initial conditions for which each differential equation below is guaranteed to satisfy Theorem 2.4.2. Do not try to solve them.

(1) $y' = \frac{x+2}{2-y}$
(2) $y' = \frac{x^3}{x+2y}$
(3) $y' = \frac{y}{x-3} + \frac{x^2}{y+4}$
(4) $y' = \frac{\tan(x)}{y^2-1}$
(5) $y' = \cot(xy)$
(6) $y' = \frac{\sqrt{y}}{x+y^3}$
(7) $y' = (xy)^{2/3}$

III. Find the general solution to each of the linear differential equations below. If an initial condition is present also find the particular solution. These problems are based on Section 2.1.

(1) $y' + xy = x$.
(2) $xy' = x^2 + 3y$.
(3) $e^x y' + 2e^x y = 1$.
(4) $3xy' - y = 1 + \ln x$, $y(1) = -2$. On what interval is your solution valid?
(5) $y' + (\tan x)y = \cos^2 x$, $y(0) = 1$. On what interval is your solution valid?

IV. Find the largest interval on which a solution to each linear initial value problem below must exist by Theorem 2.4.1. Do not try to find the solution.

(1) $y' + \dfrac{y}{t} = \dfrac{1}{t^2 - 1}$, $y(-0.5) = 4$.
(2) $y' + \dfrac{t^2 + 1}{t - 14} y = \dfrac{\sin t}{t - 4}$, $y(5) = 14,000$.
(3) $y' + \frac{1}{t^3-8} y = \frac{1}{t^2-9}$, $y(0) = 4$.
(4) $y' + \cot(t) y = tan(t)$, $y(\pi/6) = 0$.

V. Find the general solution to each of the differential equations below. Use the Bernoulli method.

(1) $y' - y = xy^2$
(2) $y' + \dfrac{y}{x} = xy^2$.
(3) $yy' + y^2 = 2x$.
(4) $y' + 3y = y^3 \sin x$.

VI. Draw the direction field by hand for the following differential equations. Use graph paper with grid locations from -5 to 5 for x and y. Do not solve these equations. See Section 1.1 for examples.

(1) $y' = 2x - 1$
(2) $y' = \frac{x}{2} + 2$
(3) $y' = y + 1$
(4) $y' = 2x + y$
(5) $y' = \frac{1}{3}(x^2 - 1)$

Math 305 | Homework Set 3 | Spring 2018

Due Monday, February 5

I. Find the general solution to each of the following differential equations. If an initial condition is given, also find the particular solution.

(1) $y' = y^3 \sin x$.

(2) $xy' + 2y = \sin x$, assume $x > 0$.

(3) $xy^2y' + y^3 = 1$, assume $x > 0$.

(4) $ty' + y = e^t$, assume $t > 0$.

(5) $y' = e^x \cos^2 y$.

(6) $y' + \dfrac{y}{t-1} = \dfrac{1}{t-3}$, assume $1 < t < 3$.

(7) $x^2y' - 3y = 2$, assume $x > 0$.

(8) $y' = \dfrac{y^2 + 2xy}{x^2}$.

(9) $y' = xy + 2y + 3x + 6$.

(10) $x^2y' + 2xy = y^3$, assume $x > 0$.

(11) $y' = xy$, $y(1) = 1$.

(12) $y' = \dfrac{x^2 + y^2}{xy}$.

(13) $z' - 3z = e^x$, with $z(0) = 2$.

(14) $y' + \dfrac{y}{2} = \sin \dfrac{x}{2}$, with $y(\pi) = 6$.

II. Use a computer to plot the direction field for each differential equation below. Use $-5 \le x \le 5$ and $-5 \le y \le 5$. Also plot two solution curves.

(1) $y' = x - 3y$.

(2) $y' = x - y^2$.

(3) $y' = \sin(\pi xy)$.

III. Do the following problems. Show all steps. Explain what you are doing.

(1) A storage tank contains 2000 gal of gasoline that initially has 100 lb of an additive dissolved in it. At $t = 0$ gasoline containing 2 lb of the additive per gallon is pumped into the tank at 40 gal/min. The gasoline in the tank is kept well mixed. It is drained out at 40 gal/min. How many pounds of the additive are dissolved in the remaining gasoline after 30 minutes?

(2) A chemical spill has polluted a pond. Your company, Clean Ponds Inc., has been contracted to clean the pond. Federal regulations require that 90% of the pollutant be removed within one month (31 days). The pond has 600,000 gallons of water in it.

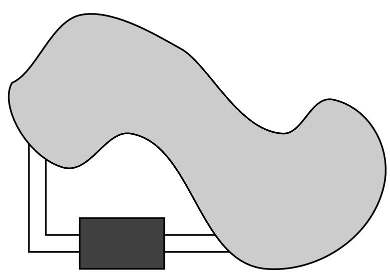
The following pump/filter systems are available:

name	Econo-Pump and Filter	Super Filter System	Pump Master with Filter
cost	\$150,000	\$250,000	\$350,000
pump rate	4000 gal/hr	3000 gal/hr	5000 gal/hr
efficiency	40 %	75 %	65 %

Notes to Table: The cost listed is the minimum rental charge per month. Filter efficiency is the percentage of pollutant removed on each pass through the filter.

A) Which pump/filter system should you get?

Note: Since the pump sends the water back into the pond after it is filtered, the water coming in is part new water and part filtered water. Assume the water in the pond is well mixed. (See figure.)



B) Because of a lawsuit by an environmental group you must get 98% of the pollutant out in one month (31 days). Now what do you do?

(3) You have a motor boat. Assume that its velocity is modeled by

$$v' = F_m - kv,$$

where F_m is the force of the motor in pounds, k is a constant, and v is velocity in feet/second. As an experiment you run your boat with various motor forcings and measure the terminal velocity. The data is in the table below.

F_m	v	k
2	4.641589	
4	7.368063	
6	9.654894	
8	11.690710	
10	13.572088	

For each experimental run compute the value of k. Is the assumed model a good one? Explain. If it is not, find a better one.

Homework Set 4

Due Monday, February 12

I. Find the solution to each of the following. You may leave your answer in the form of a relation.

(1) $\dfrac{x}{y}y' = -\dfrac{2x+y}{x+2y}$, with $y(1) = 1$.

(2) $3x^2 + y^2 + 2xyy' = 0$, with $y(1) = 2$.

(3) $(2y - x)y' = y - 4x$, with $y(2) = 1$.

(4) $\cos x \sec y + \sin x \sin y \sec^2 y \dfrac{dy}{dx} = 0$, with $y(\pi/6) = \pi/4$.

(5) $2y^2 - 6xy + (3xy - 4x^2)y' = 0$, with $y(1) = 1$. Hint: It is not exact, but will become exact if you multiply through by xy.

II.

(1) For #1 above, plot the solution curve with a computer.

III. Find the particular solution to each of the following.

(1) $y'y'' = 4t$, with $y(1) = 5$ and $y'(1) = 2$.

(2) $\frac{d^2y}{dt^2} = \frac{3}{2}y^2$, with $y(0) = 1$ and $y'(0) = 1$.

IV.

(1) At 5 a.m. you put a 60^o beer in a 40^o refrigerator. You get up at 8 a.m. expecting to have a cold one with your Honey Puffs cereal but to you horror you find that someone has taken your beer and left it on the table. The beer is now 65^o. (Room temperature, you note, is 70^o.)

You know roommate #1 left for work at 6 a.m. and so go to confront roommate #2. But, he swears that he has been asleep the whole time. You bet him a six pack you can prove it was he who left the beer out. He folds his arms, smiles and says "you're on!"

Recall Newton's Law of Cooling: $\dfrac{dT}{dt} = k(T_a - T(t))$.

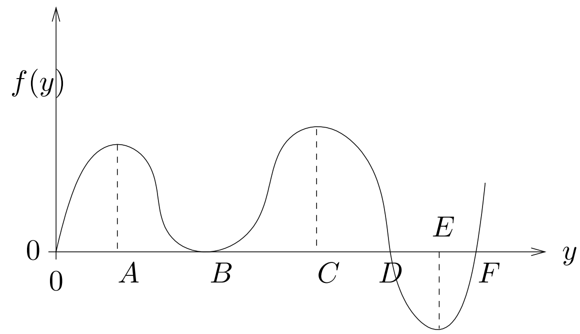
You put the beer back in the frig and wait 30 minutes (while roommate #2 watches cartoons thinking about his six pack). When you take the beer out it has cooled to 50^o. You calculate k. Now, win the bet!

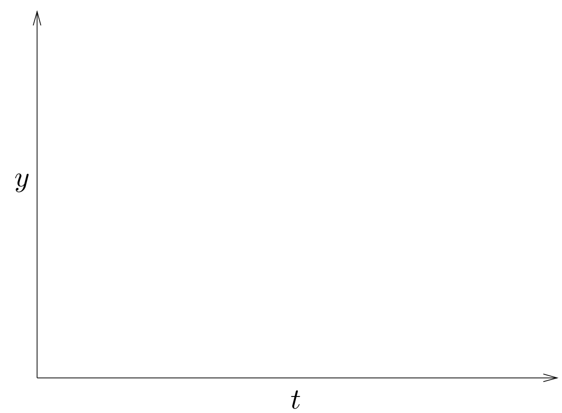
(2) The number of algae cells in a tank of water grows according to

$$A'(t) = .2\left(1 - \frac{A(t)}{100}\right)A(t) \qquad \text{light on, and}$$
$$A'(t) = -.2A(t) \qquad \text{light off.}$$

In words, the carrying capacity drops from 100 (billion cells) to zero when the light goes out. At $t = 0$ you start a ten (10) hour experimental run with $A(0) = 25$ and plan to keep the light on. When you come back at $t = 10$ you discover that the light bulb has burned out. You measure $A(10)$ to be 15. What time did the light bulb burn out?

(3) Suppose $y'(t) = F(y(t))$, where the graph of $F(y)$ is given below. Carefully draw the integral curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $y(t)$ and t are nonnegative.





Math 305 Homework Set 5 Spring 2018

Due Monday February 19

I. For each differential equation below, find the general solution, the particular solution for the initial values given, and graph it. (You can use a computer or calculator, but label the intercepts and roughly the value of any extrema.)

(1) $y'' - y' - 2y = 0$, $y(0) = 1$, $y'(0) = 2$.
(2) $4y'' - 4y' + y = 0$, $y(1) = 2$, $y'(1) = -1$.
(3) $y'' - 4y' + 13y = 0$, $y(0) = 4$, $y'(0) = -1$.
(4) $y'' + 9y = 0$, $y(0) = 2$, $y'(0) = 1$.

II. These involve equations with nonconstant coefficients.

1. Consider $t^2y'' + 4ty' - 10y = 0$. Suppose that $y(t) = t^r$ is a solution. What are the allowed values of r? What do you think the general solution is? Plug it in and check. Find the particular solution if $y(1) = 2$ and $y'(1) = 3$. On what interval do you think it will be valid?

2. Consider $y'' - \frac{1}{x}y' + 4x^2y = 0$. Assume $x > 0$. (a) Show that $y_1(x) = \sin(x^2)$ is a solution. (b) Let $y_2(x) = v(x)\sin(x^2)$. Use the reduction of order method to find another solution that is not a multiple of y_1.

3. Consider $t^2y'' - ty' + y = 0$. Suppose t^r is a solution. Find r. Use the Reduction of Order Method, as in #2, to find a second solution. What is the general solution?

III. [Extra Credit]. Consider $y' = t(5 - y)$, $y(0) = 0$. Find the solution. Use a spreadsheet program to compare the solution with the Euler Method and Improved Euler Method approximations for step-size $h = 0.1$ for $t_n = nh$, from 0.0 to 5.0.

See the spreadsheet link of the course website for an example where the equation was $y' = t - 2y$.

TEST 1

Friday, February 23

Covers up through Hwk Set 5

Math 305 Homework Set 6 Spring 2018

Due Monday March 5

I. For each differential equation below, find the general solution.

(1) $9y'' + 6y' + y = x^2$
(2) $9y'' + 6y' + y = e^{-\frac{x}{3}}$
(3) $y'' + 3y' - 4y = 2x + e^x$
(4) $y'' + 3y' - 4y = \sin x$
(5) $y'' + 4y = x$
(6) $y'' + 4y = \cos x$
(7) $y'' + 4y = \cos 2x$

II. Determine if each pair of functions is linearly dependent or independent on the given interval.

(1) $\{x^2 + x, x\}$ on $(-\infty, \infty)$
(2) $\{\sin x, \sin 2x\}$ on $(-\infty, \infty)$
(3) $\{\ln x, \ln \frac{1}{x}\}$ on $(0, \infty)$
(4) $\{e^x, e^{x+7}\}$ on $(-\infty, \infty)$
(5) $\{\frac{x+1}{x^2-1}, \frac{x}{x^2-1}\}$ on $(-1, 1)$

III. For a set of three functions, $\{f_1, f_2, f_3\}$, each twice differentiable, the Wronskian is defined to be

$$\det \begin{bmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{bmatrix}$$

A set of three functions defined on an interval I is linearly dependent if there exist real numbers, C_1, C_2 and C_3, not all zero such that

$$C_1 f_1(x) + C_2 f_2(x) + C_3 f_3(x) = 0 \qquad (*)$$

for all $x \in I$. Otherwise a set is linearly independent. It is known that if a set of three twice differentiable functions is linearly dependent then the Wronskian is zero on I. (**Extra Credit: Prove this!**)

Find the Wronskian of the four sets below. If it is not always zero conclude the set is linearly independent. If it is always zero find values for C_1, C_2 and C_3, not all zero, that satisfy $(*)$. The interval for each is the whole real line.

(1) $\{\sin^2 x, \cos^2 x, \cos 2x\}$
(2) $\{\sin x, \sin 2x, \sin 3x\}$
(3) $\{x^2 + 3x, x + 2, x^2 + x - 4\}$
(4) $\{x^2, x^2 + x, x^2 + x + 1\}$

IV. We consider differential equations of the form $y'' + p(t)y' + q(t)y = 0$. Assume $p(t)$ and $q(t)$ are continuous on the whole real line.

1. Explain why both $y_1 = t^3$ and $y_2 = t^4$ cannot be solutions.
2. Explain why both $y_1 = \sin t$ and $y_2 = t^2 + t$ cannot be solutions.

V. 1. Find a differential equation of the given form $y'' + p(x)y' + q(x)y = 0$ that has $\{x + 1, x + 2\}$ as a fundamental solution set. On what interval is it valid?

2. Find a differential equation of the given form $y'' + p(x)y' + q(x)y = 0$ that has $\{\sin x, e^x\}$ as a fundamental solution set. On what interval is it valid?

Math 305 Homework Set 7 Spring 2018

Due Monday March 19

I. For each differential equation below, find the general solution.

(1) $y'' + y = \csc t$, $0 < t < \pi$.
(2) $y'' + y = \sec t \tan t$, $-\pi/2 < t < \pi/2$.
(3) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = x^3$
(4) $y'' + 4y' + 5y = 10$.
(5) $x^2y'' + xy' - y = x$.
(6) $y' = -\frac{3x^2+y}{x+2y}$
(7) $4y'' + 4y' + y = e^{-t/2}$
(8) $y' + ty = ty^3$

II.1. A 9 lb object is attached to the lower end of a spring whose upper end is attached to the ceiling. The spring constant k is 1 lb/ft. The resistance to the motion is $\gamma = 0.25$ lb-s/ft times the velocity of the object. The object is set in motion by pulling it down 1 ft and then letting it go.

Set up a differential equation to model this mass-spring system. Solve it and plot the solution. (Don't forget to convert weight to mass.)

2. A 9 lb object is attached to the lower end of a spring whose upper end is attached to the ceiling. The spring constant k is 1 lb/ft. The resistance to the motion is $\gamma = 0.25$ lb-s/ft times the velocity of the object. The system is in equilibrium (no motion). Then, at time zero, a 1 lb hawk lands gently onto the object. Now what happens?

Set up a differential equation to model this mass-spring system. Solve it and plot the solution.

3. A 9 lb iron object is attached to the lower end of a spring whose upper end is attached to the ceiling. The spring constant k is 1 lb/ft. The resistance to the motion is $\gamma = 0.25$ lb-s/ft times the velocity of the weight. The system is in equilibrium (no motion). Then, at time zero, a oscillating magnet on the floor is turned on. It exerts a force on the object of $2\sin t$ pounds - positive is down and negative is up.

Set up a differential equation to model this mass-spring system with external forcing. Solve it and plot the solution.

III. Essay question: Look up non-linear springs. Describe several types and what they might be used for. List references you use. Where would I buy one?

Math 305 | Homework Set 8 | Spring 2018

Due Monday March 26

I. For each differential equation below, find the general solution.

(1) $xy' + 2y + 3x = 0$
(2) $4y'' + 17y' + 4y = \sin t$
(3) $y'' + 9y = 9\sec^2(3t)$, $-\pi/6 < t < \pi/6$
(4) $(e^x + 1)y' = y(1 - e^x)$
(5) $\frac{d^2H}{d\theta^2} + \frac{dH}{d\theta} - 2H = \theta e^\theta$
(6) $w'' + 9w = 9t - \cos 3t$
(7) $3y''' - 2y'' + 15y' - 10y = 0$
(8) $y'''' - y = 0$
(9) $y''' - y'' - 2y' = t^2 + t$

II.

(1) Show that $y_1(t) = \tan t$ is a solution to $y'' - (2\sec^2 t)y = 0$. Find a second solution $y_2(t)$ that is linearly independent from $y_1(t)$.

(2) Let $g(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ 1 & \text{for} \quad 0 \le t < 5 \\ 0 & \text{for} \quad t \ge 5 \end{cases}$

Let $h(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ 1 & \text{for} \quad 0 \le t < 4 \\ 0 & \text{for} \quad t \ge 4 \end{cases}$

(a) Find a differentiable solution to $y'' + \pi^2 y = g(t)$, $y(0) = y'(0) = 0$, then graph it over $t \in [0, 10]$

(b) Find a differentiable solution to $y'' + \pi^2 y = h(t)$, $y(0) = y'(0) = 0$, then graph it over $t \in [0, 10]$

(3) Find the general solution to $y'' + 3y' + 2y = \cos^2 t$. Although this is not covered by the method of undetermined coefficients given in your textbook, try $y_p = A\cos^2 t + B\sin t\cos t + C\sin^2 t$, and see what happens!

(4) Let $\epsilon(n) = \begin{cases} 1 & \text{for} \quad n = 0, 1, 4, 5, 8, 9, ... \\ -1 & \text{for} \quad n = 2, 3, 6, 7, 10, 11, ... \end{cases}$

Let $f(x) = \sum_{n=0}^{\infty} \epsilon(n) \frac{x^n}{n!}$.

Show by direct substitution that $y = f(x)$ solves the initial value problem $y'' + y = 0$, $y(0) = y'(0) = 1$.

Math 305 Homework Set 9 Spring 2018

Due Monday April 2

O. You should know how to do problems 1-27 in Section 5.1. Do not turn these in. You may be quizzed on them.

I. For each initial value problem below, find the first 5 terms of the series solution.

(1) $y'' - (\sin x)y' - 3x^2y = 0$, $y(0) = 1$, $y'(0) = 2$.
(2) $e^x y'' - 2y' + xy = 0$, $y(0) = 2$, $y'(0) = 0$.
(3) $y'' - x^2y' - 3y = 0$, $y(3) = 1$, $y'(3) = 1$.

II. Find a lower bound for the radius of convergence of the series solution to each of the following.

(1) $xy'' + (x+3)y' + \frac{y}{1+x^2} = 0$, centered at $x_0 = 4$.
(2) $y'' + \frac{1}{x-2}y' + \frac{x}{x^2+3}y = 0$, centered at $x_0 = 1$
(3) $(1+x^3)y'' + 4xy' + y = 0$, centered at $x_0 = 3$.

III. Find the general series solution to each of the following, including a recursive formula for the coefficients. If you can find a non-recursive formula do so.

(1) $y'' + xy' - y = 0$, centered at $x_0 = 0$.
(2) $xy'' + xy' + 3y = 0$, centered at $x_0 = 2$.
(3) $(x^2+1)y'' + x^3y' - y = 0$, centered at $x_0 = 0$.

IV. Prove the following identities.

(1) $[\sin t \ln(\sec t + \tan t)]' - \cos t \ln(\sec t + \tan t) = \tan t$. Assume $-\pi/2 < t < \pi/2$.
(2) $\sin^3 t = \frac{3}{4}\sin t - \frac{1}{4}\sin 3t$. Thus, if you had a $\sin^3 t$ forcing function you could convert it into a form were the method of undetermined coefficients would apply.
(3) For m and n both integers $\displaystyle\int_{-L}^{L} \cos\left(\frac{m\pi x}{L}\right)\cos\left(\frac{n\pi x}{L}\right)dx = \begin{cases} 0 & \text{for } m \neq n, \\ L & \text{for } m = n. \end{cases}$
(4) Show that for a 3×3 matrix switching the first and second rows changes only the sign of the determinant.

V. Do the following integrals.

(1) $\int \frac{1}{v^2+1}\,dv$
(2) $\int \frac{1}{v^2-1}\,dv$
(3) $\int \frac{1}{v(v+1)(v-1)}\,dv$
(4) $\int \frac{v^2}{v^2+1}\,dv$
(5) $\int \frac{v^3+v^2+v+6}{v^2-v+3}\,dv$

Math 305 Homework Set 10 Spring 2017

Due Monday, April 9

I. Find the series solution to each of the following. Find a recursive formula for a_n, and if possible, find a formula for a_n as a function of n.

(1) $y'' + y' + 2y = t^2$, centered about $t_0 = 0$.
(2) $ty'' - 3y' + y = 0$, centered about $t_0 = -1$.
(3) $y'' + (2 - t^2)y' + ty = 0$, centered about $t_0 = 0$.
(4) $y'' + xy' + (x-1)^2 y = 0$, centered about $x_0 = 1$.

II. Answer the following.

(1) If $f(x)$ is an odd function defined at $x = 0$, prove that $f(0) = 0$.
(2) Suppose $f(x)$ and $g(x)$ are even and that $h(x)$ and $k(x)$ are odd. Prove the following.
  (i) $f(x) + g(x)$ is even.
  (ii) $f(h(x))$ is even.
  (iii) $h(k(x))$ is odd.
  (iv) $h(x) \cdot k(x)$ is even.
(3) Find the average value of $\cos^2 x$ over one period.

Test 2

Friday, April 13

Covers up through Hwk Set 10

Math 305 | Homework Set 11 | Spring 2017

Due Friday, April 20

I. Find the general solution to each of the following.

1. $(e^x \sin y - 2y \sin x) + (e^x \cos y + 2\cos x)y' = 0$.
2. $t^2 y' + 2ty - y^3 = 0$.
3. $y' - 2y = 4 - x$.
4. $2y' - y = e^{x/3}$.

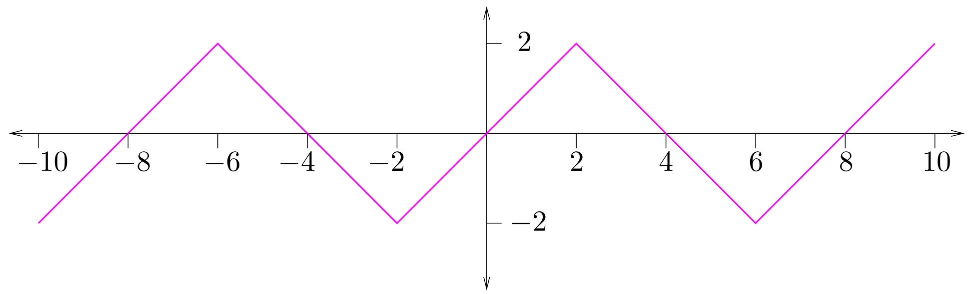
II. Find all solutions to the following boundary value problems (this is based on Section 10.1).

1. $y'' + y = 0$ with $y(0) = 0$, $y(\pi) = 0$.
2. $y'' + y = 0$ with $y(0) = 0$, $y(\pi/3) = 0$.
3. $y'' + 3y = 0$ with $y(0) = 0$, $y(\pi) = 0$.
4. For which values of γ will $y'' + \gamma y = 0$, $y(0) = 0$, $y(\pi) = 0$ have nontrivial solutions? Explain.
5. $y'' + y = 0$ with $y'(0) = 0$, $y'(\pi) = 0$.

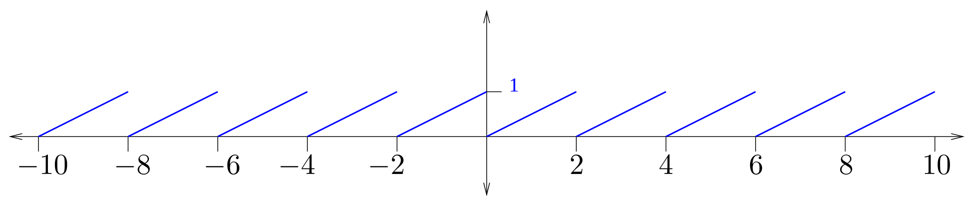
More on next page.

III. Find the Fourier series of each of the functions shown below. (Make sure you are using the correct value for L.) You can do the integration with a computer or calculator, but indicate when you have done so. Write out at least the first 10 terms.

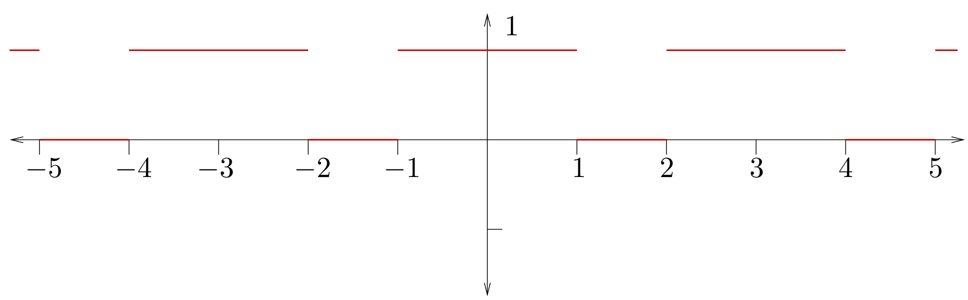
1.



2.



3.



III. For #1 above use a compute to plot the partial sums for $N = 1, 2, 5$, and 10.

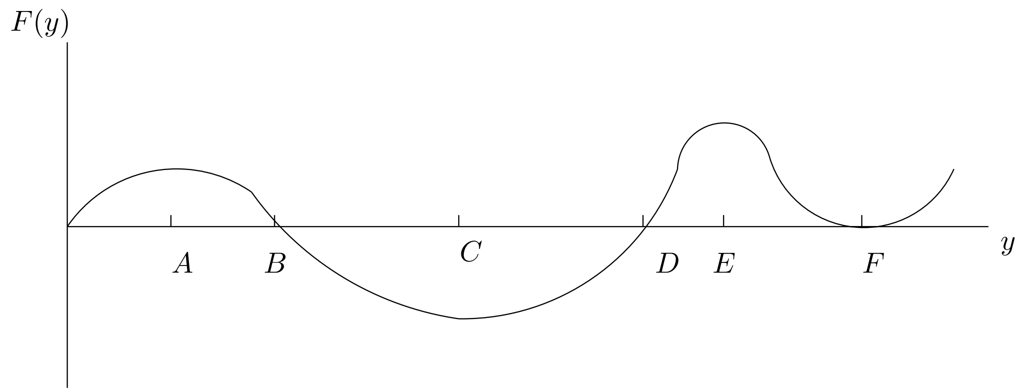
Math 305 Homework Set 12 Spring 2018

Due Friday, April 27

I. Solve the initial value problems.

1. $y'' + 4y = 3\sin 2t$, $y(0) = 2$, $y'(0) = 0$.
2. $y'' - 2y' + y = te^t + 4$, $y(0) = 1$, $y'(0) = 0$.

II. Suppose $y'(t) = F(y(t))$, where the graph of $F(y)$ is given below. Carefully draw several solution curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $y(t)$ and t are non-negative.



III. These are similar to problems 1-6 in 10.5.

1. Consider the PDE, $p(x)u_{xx} + u_t = 0$. Suppose $u(x,t) = X(x)T(t)$. Show that for some constant σ it follows that

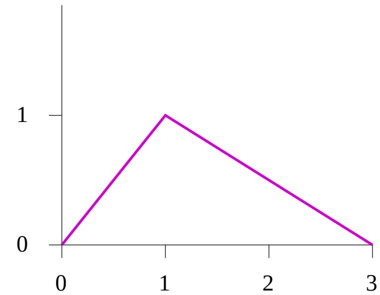
$$p(x)X'' + \sigma X = 0 \qquad T' - \sigma T = 0.$$

2. Consider the PDE, $u_{xx} + u_{yy} = u_t$. Suppose $u(x,y,t) = X(x)Y(y)T(t)$. Show that for some constants α and σ it follows that

$$T' + \sigma T = 0 \qquad X'' + \alpha X = 0 \qquad Y'' + (\sigma + \alpha)Y = 0.$$

IV. Now for the fun stuff.

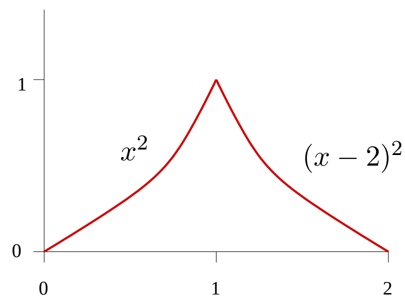
1. Consider a vibrating ideal string (that is assume the wave equation is valid) with $a = 1$, $L = 3$, with initial displacement $f(x)$ given below.



Find $u(x,t)$. Plot the approximation of $u(x,t)$ using 30 terms for $t = 0.0$, $t = 0.5$, $t = 1.0$, $t = 1.5$, and $t = 2.0$.

2. Consider a vibrating ideal string (that is assume the wave equation is valid) with $a = 1$, $L = \pi$, with initial displacement $f(x) = \sin 5x$. Find $u(x,t)$ and plot it for $t = 0.0$, $t = 0.1$, $t = 0.2$, $t = 0.3$,$t = 0.4$ and $t = 0.5$.

3. Consider a metal rod of length 2 and $\alpha = 1$ with the initial temperature shown below. Assume the end points are held to by 0^o.



Find $u(x,t)$. Plot the first thirty terms of $u(x,t)$ for $t = 0.0$, $t = 0.01$, $t = 0.03$, $t = 0.1$, and $t = 0.5$.

4. Animate any of these for extra credit. You can just show me the animation on your laptop/tablet or email it to me.