Math 305 Quiz 2 Spring 2018

Name: Key

ONLY SCIENTIFIC CALCULATORS ALLOWED

1. [10 points] A mass-spring system has mass $m = 2$, damping $\gamma = 2$, and spring constant $k = 7$. (Don't worry about the units.) There is no external forcing.

   a. Write down the differential equation for this system.

   $$2u'' + 2u' + 7u = 0$$

   b. What is the general solution?

   $$2r^2 + 2r + 7 = 0$$

   $$r = \frac{-2 \pm \sqrt{4 - 4 \cdot 2 \cdot 7}}{4} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{-13}$$

   $$= -\frac{1}{2} \pm \frac{\sqrt{13}}{2} i$$

   $$u(t) = C_1 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{13}}{2}t\right) + C_2 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{13}}{2}t\right)$$

   c. Is this system under damped, over damped or critically damped?

2. [10 points] Consider $y'' - xy' + x^2y = 0$ with initial conditions $y(0) = 1$ and $y'(0) = 3$. Assume that the solution has a Taylor series:

$$y(x) = \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} \frac{y^{[n]}(0)}{n!} \cdot x^n.$$

Find the first five terms: a_0, a_1, a_2, a_3, a_4.

$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$

$y(0) = a_0$. Since $y(0) = 1$, we have $a_0 = 1$.

$y'(0) = a_1$. Since $y'(0) = 3$, we have $a_1 = 3$.

minus

$y'' = xy' \cancel{+} x^2 y$

$y''(0) = 0$ Hence $a_2 = \frac{0}{2!} = 0$.

$y''' = y' + xy'' - 2xy - x^2y'$

$y'''(0) = 3 + 0 - 0 - 0 = 3$ $\quad a_3 = \frac{y'''(0)}{3!} = \frac{3}{3!} = \frac{1}{2}$

$y'''' = y'' + y'' + xy''' - 2y - 2xy' - 2xy' - x^2y''$

$y''''(0) = \underbrace{2y''(0)}_{=0} + 0 - \underbrace{2y(0)}_{=1} - 0 - 0 - 0 = -2$

$a_4 = \frac{y''''(0)}{4!} = \frac{-2}{4!} = \frac{-1}{12}$.

Thus,

$y(x) \approx 1 + 3x + \frac{1}{2}x^3 - \frac{1}{12}x^4$