Partial Derivatives for Math 305

Since partial derivates are not covered in Calculus II (Math 250), but the textbook for Math 305 assumes you have had this, I made this handout. If you have had Calculus III (Math 251) or have seen partial derivates elsewhere you shouldn't need to read this.

Let $f(x, y)$ be a function of two variables. The **partial derivative** of f with respect to x measures the rate of change in the value of $f(x, y)$ as x varies, but y is held fixed. There are several common notations for this:

$$\frac{\partial f}{\partial x} \qquad \partial_x f \qquad f_x$$

all mean the partial derivative of f with respect to x.

In pratice this is easy to compute, you pretend y is a constant and find the derivate in usual way. Here are some examples.

1. Let $f(x, y) = x^2y^3$. Then $\frac{\partial f}{\partial x} = 2xy^3$.
2. Let $f(x, y) = x \sin xy$. Then $\frac{\partial f}{\partial x} = \sin xy + xy \cos xy$. Here we used the product rule and the chain rule. If you are confused, replace y with the number 3 or the letter a and compute the derivative as usual.
3. Let $g(t, z) = \tan t^2z + \sinh z^3$. Then $\frac{\partial g}{\partial t} = 2tz \sec^2 t^2z$.

The partial derivate of $f(x, y)$ with respect to y is defined similary as the rate of change of f as y varies and x is held fixed. Here are some examples.

4. Let $f(x, y) = x^2y^3$. Then $\frac{\partial f}{\partial y} = 3x^2y^2$.
5. Let $f(x, y) = x \sin xy$. Then $\frac{\partial f}{\partial y} = x^2 \cos xy$. Only the chain rule was needed.
6. Let $g(t, z) = \tan t^2z + \sinh z^3$. Then $\frac{\partial g}{\partial z} = t^2 \sec^2 t^2z + 3z^2 \cosh z^3$.

You can refer to Section 11.3 of the calculus textbook used at SIU for more. Any calculus textbook will have a section on partial derivatives.