Small Damping Approximations

In Sections 3.8 and 3.9 the text uses certain standard approximations for small values of the damping coefficient γ. See page 177, equation 26; page 178 equation 27; and page 187, equation 13. The first of these (page 177) is derived from equation 1 below while the other two are derived from equation 2 below. The text's results can be recovered by setting $x = \gamma^2/(4km)$ into the appropriate equation below.

$$\sqrt{1-x} \approx 1 - \frac{1}{2}x \tag{1}$$

$$\frac{1}{\sqrt{1-x}} \approx 1 + \frac{1}{2}x \tag{2}$$

To get equation 1, just look at the Taylor series for $f(x) = \sqrt{1-x}$ centered about $x = 0$. The derivative is

$$f'(x) = \frac{-1}{2\sqrt{1-x}}.$$

Thus, $f(0) = 1$ and $f'(0) = -1/2$. Hence the first two terms of the Taylor series of $f(x)$ are $1 - (1/2)x$. Equation 1 is now justified. Equation 2 is done similarly.

It is instructive to plug in values of x and see how good or bad these approximations are.

x	.001	.01	.1	.2	.3
$\sqrt{1-x}$	.999499875	.994987437	.948683298	.894427191	.836660027
$1 - \frac{1}{2}x$	.9995	.995	.95	.9	.85
$\frac{1}{\sqrt{1-x}}$	1.000500375	1.005037815	1.054092553	1.118033989	1.195228609
$1 + \frac{1}{2}x$	1.0005	1.005	1.05	1.1	1.15