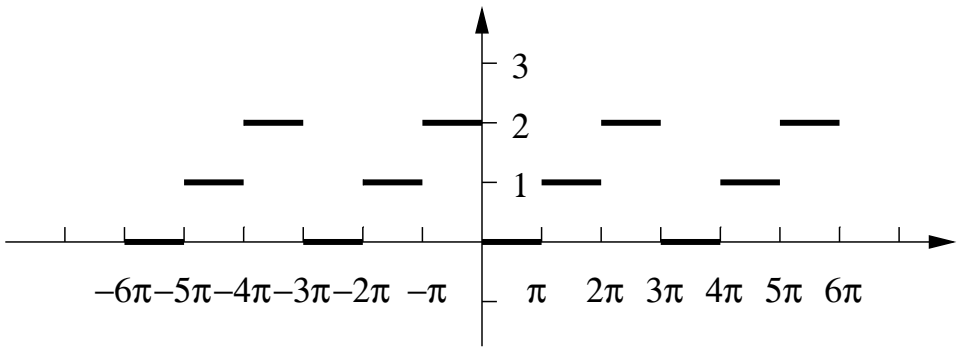
Practice Final Exam, Math 305, Spring Semester, 1997

1. Let $f(x)$ be a periodic function defined by the graph below.



   a. Find a_0.

   b. Find b_3.

2. Consider a metal rod, 1 foot long. Let the initial temperature distribution be given by $f(x) = 0$. Now suppose the ends are somehow set to be

$$u(0,t) = 10^o \quad \text{and} \quad u(1,t) = 20^o,$$

   for $t > 0$. Write down all of the integrals you would need to solve this problem AND show how you would put the results together to obtain $u(x,t)$. DO NOT EVALUATE ANY OF THE INTEGRALS!

3. Let $y'' + xy' + 3y = 0$. Use the Taylor series method, centered about $x = 0$ to find the general solution. You must find the recursive formula for a_n.

4. Draw the direction field of $y' = \dfrac{3-y}{2}$.

5. a. Solve $\dfrac{dy}{dx} = \dfrac{x^3 - 2y}{x}$.

   b. Solve $\dfrac{dy}{dx} = e^{2x} + 3y$.

6. The Gompertz equation is used in population dynamics. It is

$$\frac{dN}{dt} = rN \ln\left(\frac{K}{N}\right).$$

a. Find $\lim_{N\to 0} \dfrac{dN}{dt}$.

b. What is $\dfrac{dN}{dt}$ when $N = K$?

c. Graph $\dfrac{dN}{dt}$ vs N for $0 < N \leq K$.

d. Find the location and value of the maximum of this graph.

e. Sketch the graph of several solution curves, N vs t, for $t > 0$.

f. Indicate the concavity of the solution curves.

g. What is $\lim_{t\to\infty} N(t)$?

7. For an object moving under the influence of the Earth's gravity we have

$$m\frac{dv}{dt} = -\frac{mgR^2}{(R+x)^2},$$

where R is the Earth's radius and x is the height above the earth. Solve this differential equation. Obtain v as a function of x and the objects initial velocity, v_0. Resolve any ambiguity in the signs.

8. Solve the initial valve problem

$$y'' + y' - 2y = e^x,$$

$$y(0) = 0 \text{ and } y'(0) = 0.$$

9. a. A 20g mass stretches a spring 5 cm. Find the spring constant K.

b. Let $\gamma = 400$ be the damping constant. We pull the mass down 2 cm more and then let go ($u(0) = 2, u'(0) = 0$). Find $u(t)$.

c. About how many oscillations will there be until the amplitude is below .1 cm?