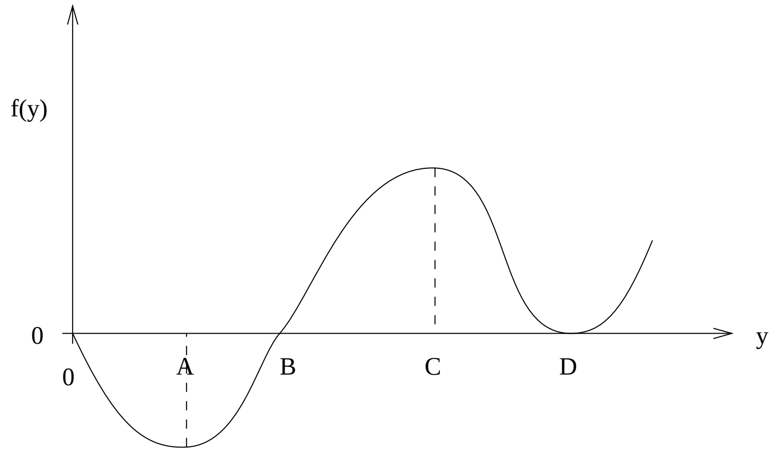
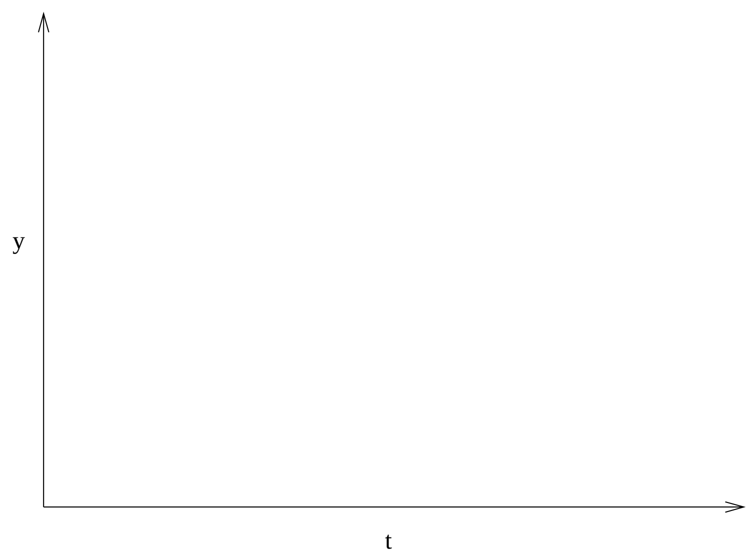
Test 1 **Math 305** **Spring Semester, 1997**

Each problem is worth 20 points.

1. Suppose $N'(t) = F(N(t))$, where the graph of $F(N)$ is given below. Carefully draw the integral curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $N(t)$ and t are nonnegative.





2. Solve the initial value problem, $(x^2+1)y' + 3xy = 6x$, with initial condition $y(0) = 2$.

3. Find the general solution of the differential equation $(x^2+y)y' + 2xy = 6x$. Hint: check for exactness. (You need not solve for y.)

4. Find the general solution of the equation below. (You need not solve for y.)

$$(x^2 y + yx - y)\,dx \;+\; x^2(y-2)\,dy = 0$$

5. The equation below is homogeneous (in the sense of section 2.9). Find its general solution. (You need not solve for y.)

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

6. **BONUS PROBLEM.** The number of algae cells in a tank of water grows according to

$$\begin{aligned} A'(t) &= .2\left(1 - \frac{A(t)}{100}\right)A(t) && \text{light on, and} \\ A'(t) &= -.2A(t) && \text{light off.} \end{aligned}$$

In words, the carrying capacity drops from 100 (billion cells) to zero when the light goes out. At $t = 0$ you start a ten (10) hour experimental run with $A(0) = 25$ and plan to keep the light on. When you come back at $t = 10$ you discover that the light bulb has burned out. You measure $A(10)$ to be 15. What time did the light bulb burn out?

Hints: You can use your graphing calculator, but make sure your answer in correct to at least 4 decimal places. The integral below will be helpful:

$$\int \frac{dx}{x(a + bx)} = \frac{1}{a} \ln\left(\frac{x}{a + bx}\right) + C$$