Test 2 **Math 305** **Spring Semester, 1997**

Each problem is worth 20 points.

1. a) Find the general solution of $y'' - y = \sin x$.

   b) Suppose $y(0) = 1$ and $y'(0) = 1$. Does the $\sin x$ term have a significant impact on the long term ($x \to \infty$) of the behavior of the system. EXPLAIN.

   c) Find a pair of initial conditions such that the long term ($x \to \infty$) behavior of the system is approximately periodic.

2. What is the radius of convergence for the power series solution of $(x^2+1)y''+xy'+3y = 0$, centered about $x_0 = 1$? (Do not try to find the solution!)

3. Let $y_1(x)$ and $y_2(x)$ be linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$. Suppose that $y_1(a) = y_1(b) = 0$. Show that $y_2(x)$ has a zero in the open interval (a, b). Assume $p(x)$ and $q(x)$ are continuous everywhere.

4. The differential equation of an unforced damped mass-spring oscillator is, of course, $mu'' + \gamma u' + ku = 0$. Derive a formula for the critical damping value of γ. Recall that critical damping occurs when γ is the smallest value that does not give complex roots of the characteristic equation.

5. Find the first 4 terms of the power series solution to $2y'' + (x + 1)y' + 3y = 0$, with $y(0) = 1$ and $y'(0) = 1$, centered at $x_0 = 0$.

6. Show that x and x^2 solve the equation $\frac{1}{2}x^2y'' - xy' + y = 0$. Now consider $\frac{1}{2}x^2y'' - xy' + y = x$. Use the variation of parameters method to find the general solution. Assume $x > 0$.