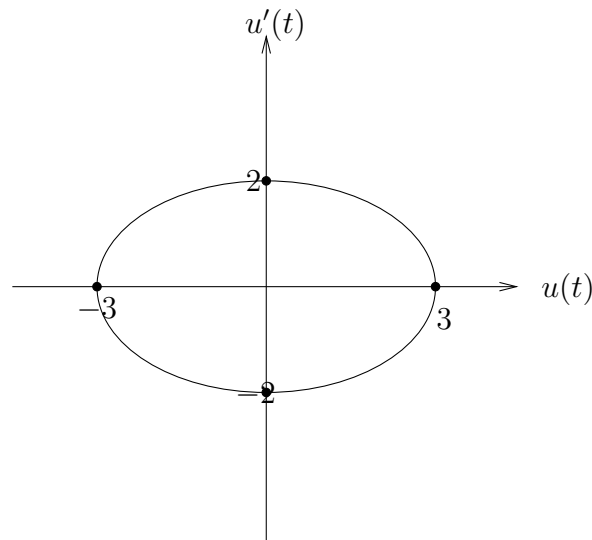


Each problem is worth 20 points.
NO CALCULATORS

1. Find the solution set of $y'' - 2y' + 1 = e^x$.
2. Consider the third order equation, $y''' - 4y'' + 5y' - 2y = 0$. Find the characteristic equation and show that its **only** roots are $r = 1$ and $r = 2$. Find three linearly independent solutions. Verify your results by direct substitution.
3. Consider the initial value problem $y'' + y = g(t)$ with $y(0) = y'(0) = 0$ where $g(t)$ is one for $t \in [\pi, 2\pi]$ and zero elsewhere. Find a differentiable solution $y(t)$. Graph it for $t \in [0, 6\pi]$.
4. Below is the phase portrait of an undamped oscillator. Find its natural frequency.



5. Suppose that x^3 and x are solutions to $y'' + py' + qy = 0$.
 - a) Find $p(x)$. Hint: use Abel's formula for the Wronskian.
 - b) Set $y(x) = x$. Substitute this into the differential equation, using the $p(x)$ from part (a), and solve for $q(x)$.
 - c) Use the $p(x)$ and $q(x)$ from parts (a) and (b) and show that $y(x) = x^3$ is indeed a solution.
6. We throw a ball, from ground level, up in the air, with an initial velocity of 5 m/s. Assume the force of the air resistance is proportional to the magnitude of the velocity; call the proportionality constant k .

Write down the different equation for the velocity $v(t)$.

Solve it for $v(t)$.