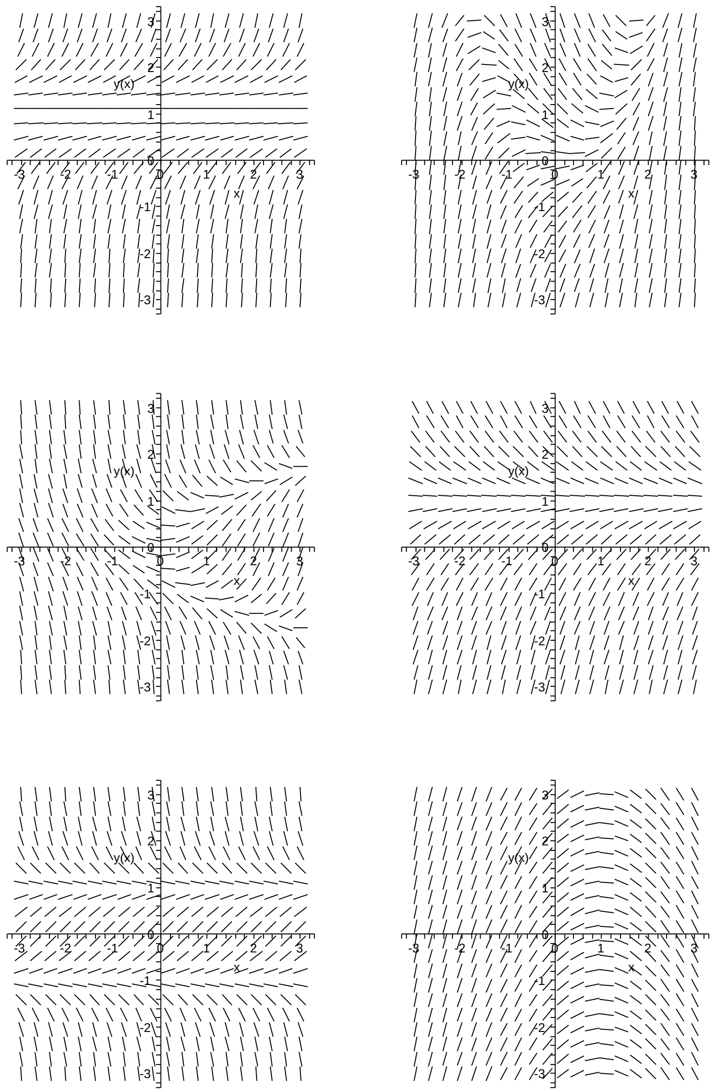
Math 305 Final Exam Spring 1999

Name: ______________________ ID #: ______________________

Part I: NO CALCULATORS

1. [20 points] Match the differential equation with its direction field. (You get 4 points for each correct match, -1 for each wrong match.)

   (1) $y' = y - x$, (2) $y' = 2 - y$, (3) $y' = |y - 2|$, (4) $y' = y + x$, (5) $y' = x - y$.



2. [20 points] Solve the initial value problem, $(x^2 + 1)y' + 3xy = 6x$, with initial condition $y(0) = 2$.

3. [20 points] Consider the equation

$$y' = \frac{y}{x-1}.$$

Find the general solution. Draw the integral "curves" for the following initial values: $y(0) = \pm 2, \pm 1, 0$, and $y(2) = \pm 2, \pm 1, 0$. Hint: be careful with your absolute value signs.

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Part II: CALCULATORS ALLOWED

4. [20 points] A body of mass m falls from rest in a medium offering resistance proportional to the square of the velocity. Find the relation between the velocity v and the time t. Find the limiting velocity, v_l.

   Hint:
$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{1}{2ab}\ln\left|\frac{a+bx}{a-bx}\right| + C,$$
   where a and b are positive constants.

5. [20 points] Find the general solution of $y'' + 2y' + y = \cos(\alpha x)$.

6. [20 points] Suppose that $y = f(x)$ and $y = g(x)$ are linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$. Suppose further that it is known that the Wronskian of f and g is 1 for all values of x. Find $p(x)$. Hint: Use Abel's formula.

7. [20 points]

   a. A 20g mass stretches a spring 5 cm. Find the spring constant K, in g/sec^2. [$g = 980$ cm/sec^2]

   b. Let $\gamma = 40$ dyne-sec/cm be the damping constant. We pull the mass down 2 cm more and then let go ($u(0) = 2, u'(0) = 0$). Find $u(t)$.

   c. Graph $u(t)$. About how many oscillations will there be until the amplitude is below .5 cm?

8. [20 points] Consider the 3rd order differential equation, $y''' + (x+1)y'' + (\sin x)y' + y = 0$, with initial conditions $y(0) = 0$, $y'(0) = 1$, and $y''(0) = 2$. Apply the series method and find the first 5 terms of the Taylor series for $y(x)$, centered about zero.

9. [20 points] Consider a metal rod, 1 foot long. Let the initial temperature distribution be given by $f(x) = 0$. Now suppose the ends are somehow set to be
$$u(0,t) = 10^o \quad \text{and} \quad u(1,t) = 20^o,$$
   for $t > 0$. Write down all of the integrals you would need to solve this problem AND show how you would put the results together to obtain $u(x,t)$. DO NOT EVALUATE ANY OF THE INTEGRALS!

10. [20 points] Let
$$f(x) = \begin{cases} 1 & x \in [0,1] \\ x & x \in [1,2] \\ 2 & x \in [2,3] \end{cases}.$$
    Graph the even and odd extensions of $f(x)$. Find the first two terms in of the Fourier series of the even extension.

Hints:

$$\int x\cos(ax)\,dx = \frac{\cos(ax) + ax\sin(ax)}{a^2} + C$$

$$\int x\sin(ax)\,dx = \frac{\sin(ax) - ax\cos(ax)}{a^2} + C$$