

Name: \_\_\_\_\_ ID #: \_\_\_\_\_

1. [15 points] Consider the differential equations below:

$$y'' + p(x)y' + q(x)y = 0 \quad (1)$$

$$y'' + p(x)y' + q(x)y = g(x) \quad (2)$$

Suppose that  $y = f(x)$  solves (1) and that  $y = h(x)$  solves (2). Only one of the statements below is always true. Which one is it? Prove it.

- a.  $f(x) + Ch(x)$  solves (1)
  - b.  $f(x) + Ch(x)$  solves (2)
  - c.  $h(x) + Cf(x)$  solves (2)
2. [10 points] Prove that  $e^{rx}$  and  $xe^{rx}$  are linearly independent.
3. [15 points] Prove that if  $x$  and  $x^2$  are solutions to  $y'' + py' + qy = 0$  then  $p(x)$  and  $q(x)$  cannot both be continuous at  $x = 0$ .
4. [20 points] Consider  $2t^2y'' + 3ty' - y = 0$ . Show that  $y = 1/t$  is a solution. Then find a second solution of the form  $y = v(t)/t$ .
5. [20 points] Solve  $y'' + 2y' + y = \sin x$ , with initial conditions  $y(0) = y'(0) = 0$ .
6. [20 points] The differential equation of an unforced damped mass-spring oscillator is, of course,  $mu'' + \gamma u' + ku = 0$ . Derive a formula for the critical damping value of  $\gamma$ . Recall that critical damping occurs when  $\gamma$  is the smallest value that does not give complex roots of the characteristic equation.