Math 305 Final Exam Fall 2001

Name: _____ ID #: _____

Scientific Calculators Only

1. [20 points] Solve the following for y. Find the exact value of the integration constant.

   a) $y' = -y \ln\left(\frac{1}{y}\right)$, $y(0) = e^e$.

   b) $y' = \cos^2(y/x) + y/x$, $y(1) = \pi/4$.

2. [20 points] Find the general solution to each of the following. Solve for y.

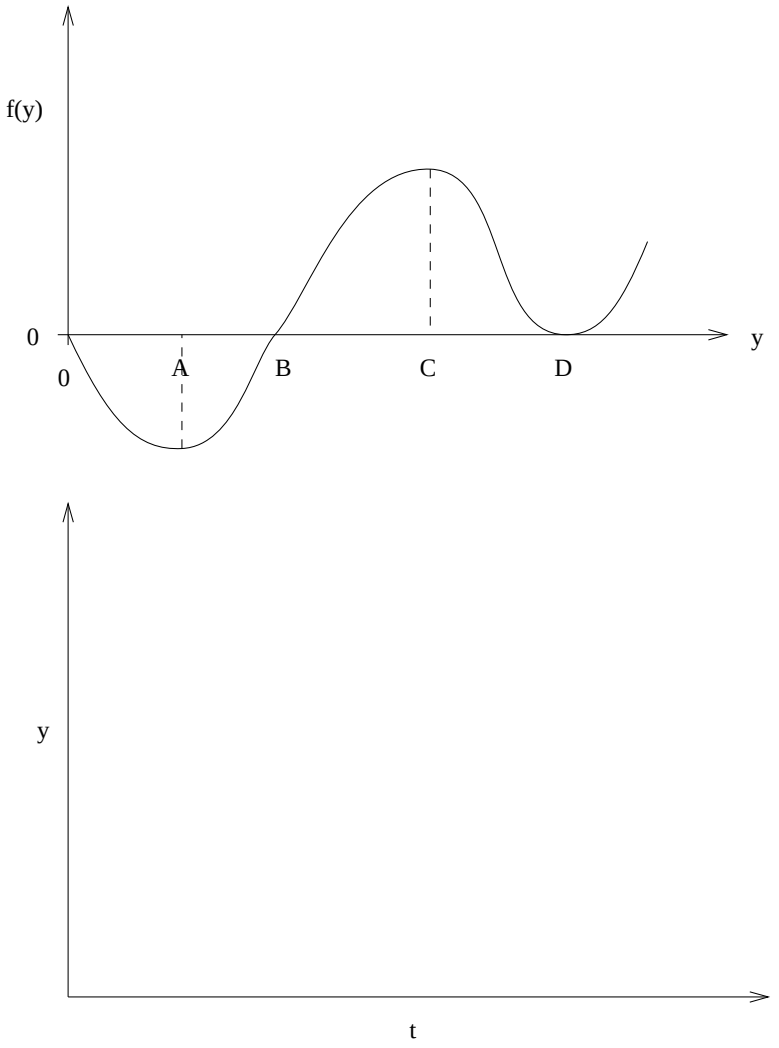
   a) $(3xy + y^2) + (x^2 + xy)y' = 0$.

   Hint: use $\mu = x$ as an integrating factor, then check for exactness.

   b) $t^2y' + 2ty = y^3$, $t > 0$.

   Hint: This is a Bernoulli equation; use the substitution $v = 1/y^2$.

3. [20 points] Suppose $N'(t) = F(N(t))$, where the graph of $F(N)$ is given below. Carefully draw the integral curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $N(t)$ and t are non-negative.



4. [20 points] A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ grams/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount if salt in the tank as $t \to \infty$.

5. [20 points]

   A body of mass m falls from rest in a medium offering resistance proportional to the square of the velocity. Find the relation between the velocity v and the time t. Find the limiting velocity, v_l.

   Hint:
   $$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \ln \left| \frac{a + bx}{a - bx} \right| + C.$$

6. [20 points] Find the general solution to $y'' - 3y' - 10y = 3e^{5t} + 2e^{6t} + \sin(2t)$.

7. [20 points] Find the solution to $y'' - y' - 7y = t + 1$, $y(0) = y'(0) = 0$.

8. [20 points] Find the general series solution to $y'' - xy' - y = 0$ centered about $c = 1$. You need to find the recursive formula for the a_n's.

9. [20 points] Let

$$f(x) = \begin{cases} 1 & x \in [0,1] \\ x & x \in [1,2] \\ 2 & x \in [2,3] \end{cases} .$$

Graph the even and odd extensions of $f(x)$. Find the first two terms in of the Fourier series of the even extension.

Hints:

$$\int x\cos(ax)\,dx = \frac{\cos(ax) + ax\sin(ax)}{a^2} + C$$

$$\int x\sin(ax)\,dx = \frac{\sin(ax) - ax\cos(ax)}{a^2} + C$$

10. [20 points] Consider a metal rod, 1 foot long. Let the initial temperature distribution be given by $f(x) = 0$. Now suppose the ends are somehow set to be

$$u(0,t) = 10^o \quad \text{and} \quad u(1,t) = 20^o,$$

for $t > 0$. Write down all of the integrals you would need to solve this problem AND show how you would put the results together to obtain $u(x,t)$. DO NOT EVALUATE ANY OF THE INTEGRALS!