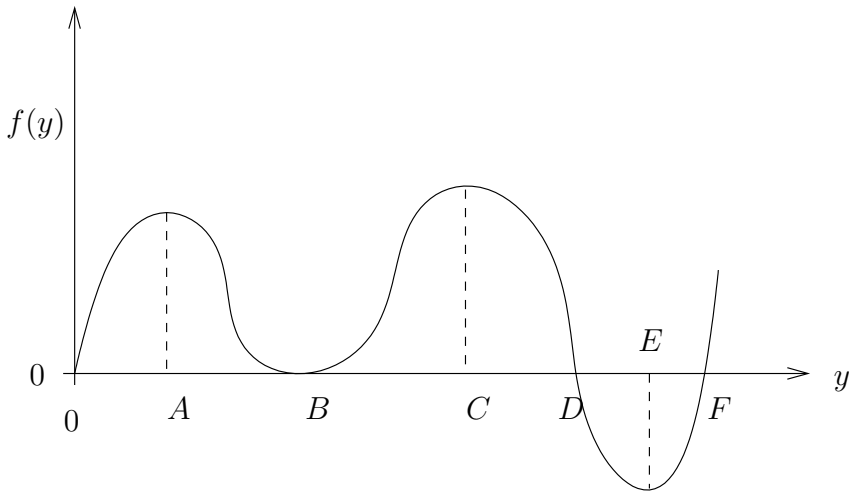
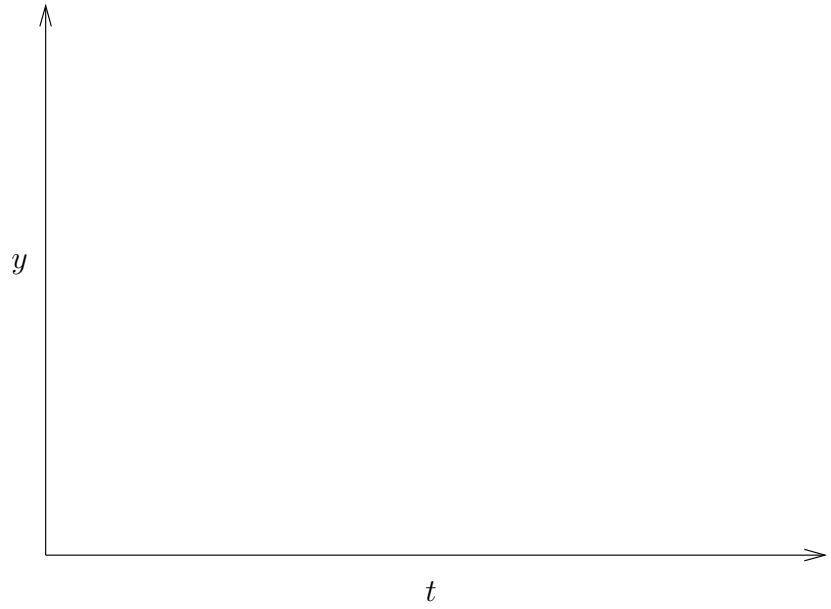
Math 305 Test 1 Version β Fall 2001

Name: _____ ID #: _____

SCIENTIFIC CALCULATORS ONLY

1. [10 points] Suppose $y'(t) = F(y(t))$, where the graph of $F(y)$ is given below. Carefully draw the integral curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $y(t)$ and t are nonnegative.





2. [10 points] On the attached sheets there are six direction fields. Match each of the five equations below with a direction field. Each correct match is worth 2 points, each incorrect match is -1 point.

(a) $y' = 1 - y^2$

(b) $y' = (1 - y)^2$

(c) $y' = y - x$

(d) $y' = x - y$

(e) $y' = x + y$

3. [20 points] A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ grams/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount if salt in the tank as $t \to \infty$.

4. [40 points] Solve the following for y. Find the exact value of the integration constant.

a) $y' = -y \ln\left(\frac{1}{y}\right), \qquad y(0) = e^e.$

b) $y' - (\sin t)y + \sin t = 0, \qquad y(0) = 2.$

c) $3x^2y^2 + 3y + (2x^3y + 3x + 2)y' = -1, \qquad y(0) = 2.$

d) $y' = \cos^2(y/x) + y/x, \qquad y(1) = \pi/4.$

5. [20 points] Consider the differential equation $y' + y/t = y^3$. The methods we have covered do not work. But here you will learn how to to this. Let $v = 1/y^2$.

Step 1. Find dy/dt in terms of v and dv/dt.

Step 2. Transform $y' + y/t = y^3$ into a differential equation involving v', v and t only.

Step 3. Simplify this equation, and show it is now linear.

Step 4. Solve it for $v(t)$. [When computing μ be sure to simplify before integrating.]

Step 5. Solve for $y(t)$.