Math 305 Test 2 Fall 2001

SCIENTIFIC CALCULATORS ONLY

1. [15 points] a) Solve $y'' + 4y' + 4y = 0$, $y(0) = 0$, $y'(0) = 2$.

   b) Now consider $y'' + 4y' + 4y = e^{-2t} + t^2$. What is the best "guess" for y_p? [Do not try to find the coefficients.]

2. [15 points] a) Find the general solution to $y'' + 2y' + 3y = e^{-t}$.

   b) What is the limit as $t \to \infty$?

3. [25 points] Solve $y'' + y = F(t)$, $y(0) = 0$, $y'(0) = 0$, where

$$f(t) = \begin{cases} 0 & t < 0 \\ t & 0 \le t \le \pi \\ 1 & \pi < t. \end{cases}$$

   (Your solution and its derivative should be continuous.)

4. [25 points] A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1in., and then set in motion with a downward velocity of 2 ft/sec, and if there is no damping, find the position u of the mass at ant time t. Determine the frequency, period, amplitude, and phase of the motion.

5. [20 points] Which of the pairs of functions below are Linearly Independent? Justify your answer.

   a) e^x, e^{2x} b) $\sin x$, $\cos x$.

   c) $\ln x$, $\ln x^3$ $(x > 0)$. d) $2t^2 - 2$, $t^3 + 3t$

6. [20 bonus points] Consider $y'' + p(t)y' + q(t)y = 0$. Prove that if y_1 and y_2 are solutions that have a common point of inflection t_0 in an open interval I where p and q are continuous, then they cannot be a fundamental set of solutions on I unless both p and q are zero at t_0.