Math 305 Test 3 Fall 2001

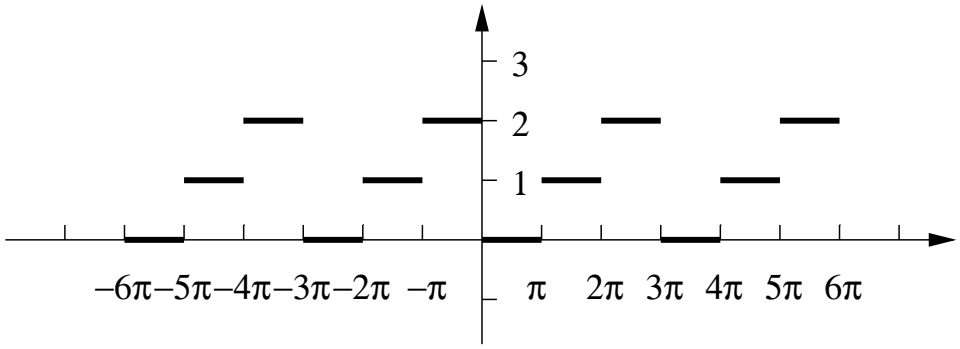
SCIENTIFIC CALCULATORS ONLY

1. [20 points] Consider $y'' + 2\sin(x)y' - 3y = 0; y(0) = 0, y'(0) = 2$. Find the first **four** terms of the power series of the solution.

2. [20 points] Consider $y'' + 2xy' - 3y = 0$. Find the general series solution; in particular find a recursive formula for the a_n's.

3. [20 points] Consider $(x^2 + 4)y'' + (x^3 + 1)y' - 4x^2y = 0$. DO NOT TRY TO SOLVE THIS. Merely find a lower bond on the radius of convergence of the series solution centered about $c = 1$.

4. [20 points] Let $f(x)$ be a periodic function defined by the graph below.



   a. Find a_0.

   b. Find b_5.

5. [20 points] Consider the partial differential equation

$$U_{xx} - U_{xt} - U_t = 0.$$

   Suppose that there is a solution of the form $U(x,t) = X(x)T(t)$. Show that $X(x)$ and $T(t)$ must satisfy the ordinary differential equations below:

$$X'' + \sigma X' + \sigma X = 0$$

$$T' + \sigma T = 0$$

6. [20 bonus points] Let $f(x)$ be an even periodic function with period $2L$. (Thus, the b_n coefficients of its Fourier series are all zero.) If the function enjoys the additional symmetry $f(x) = -f(L - x)$ it can be shown that for even values of n, $a_n = 0$.

   a. Prove that $a_0 = 0$. Hints: Break up the integral $\dfrac{2}{L}\displaystyle\int_0^L f(x)\,dx$ at $L/2$. The substitution $u = L - x$ may be helpful at a certain point.

   b. Prove the general case, $a_n = 0$ for n even.