Print Name: _____

Math 305 **Test 1** **Scientific Calculators Allowed**

1. [10 points] You are giving presentation at your new job. In the midst of it someone points out that your direction field slides do not match your equations. You realize that your secretary has mixed the slides up! Quickly figure out which direction field matches each equation. (Your secretary also included an extra one you do not need.) Each correct match is worth 2 points, each incorrect match is -1 point.

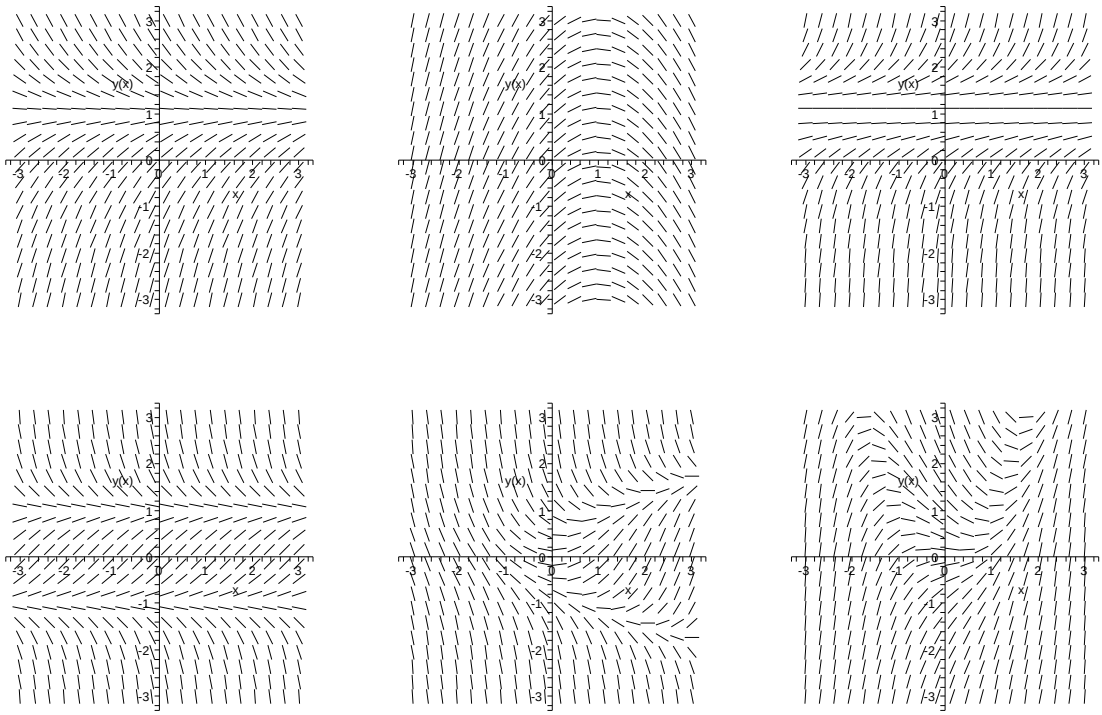
   (a) $y' = 1 - y^2$

   (b) $y' = (1 - y)^2$

   (c) $y' = y - x$

   (d) $y' = x - y$

   (e) $y' = x + y$



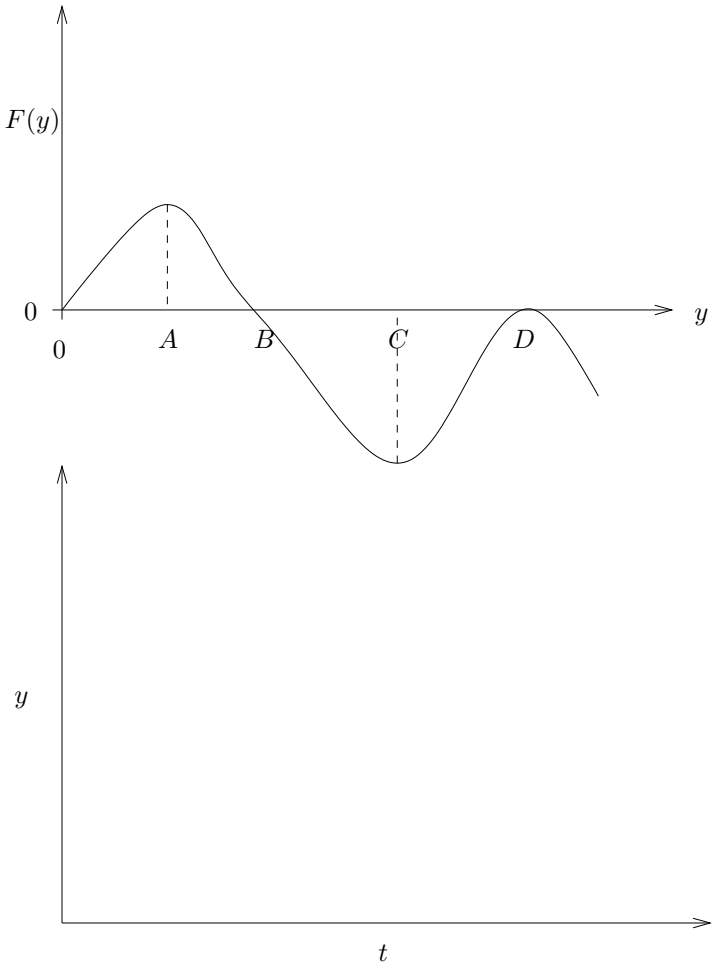
2. [15 points] Suppose $y'(t) = F(y(t))$, where the graph of $F(y)$ is given below. Carefully draw the integral curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $y(t)$ and t are nonnegative.

3. [15 points] A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ grams/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount if salt in the tank as $t \to \infty$.

4. [15 points] Solve the initial value problem $y' - (\sin t)y + \sin t = 0$, with $y(0) = 2$. Express y as a function of t and find the exact value of the integration constant. Simplify your answer.

5. [15 points] Show that the differential equation $(3x^2y^2 + 3y + 2) + (2x^3y + 3x + 1)y' = 0$, is exact, then find the general solution. Do not try to solve for y.

6. [15 ponts] Solve $y'' + 6y' - 7y = 0$, with $y(0) = 2$ and $y'(0) = 3$.



7. [15 points] Solve the initial value problem $y' = \cos(y/x) + y/x$, with $y(1) = 0$. Do not try to solve for y but simplify as much as you can. Hint: $\int \sec u \, du = \ln|\sec u + \tan u| + C$.

8. **BONUS PROBLEM.** [10 points] Find the general solution to $y' + y/t = y^3$. Hint: This is a Bernoulli equation, let $v = 1/y^2$.

9. **BONUS PROBLEM.** [10 points] The number of algae cells in a tank of water grows according to

$$\begin{aligned} A'(t) &= .2\left(1 - \frac{A(t)}{100}\right)A(t) && \text{light on, and} \\ A'(t) &= -.2A(t) && \text{light off.} \end{aligned}$$

In words, the carrying capacity drops from 100 (billion cells) to zero when the light goes out. At $t = 0$ you start a ten (10) hour experimental run with $A(0) = 25$ and plan to keep the light on. When you come back at $t = 10$ you discover that the light bulb has burned out. You measure $A(10)$ to be 15. What time did the light bulb burn out?

Hint: This integral will be helpful: $\displaystyle\int \frac{dx}{x(a+bx)} = \frac{1}{a}\ln\left(\frac{x}{a+bx}\right) + C.$