Print Name: ________________________________ ID number: ________________________________

Fall 2004 **Math 305** **Test 2**

Only Scientific Calculators Allowed

1. [10 points] Consider $y'' + 2\sin(x)y' - 3y = 0$, $y(0) = 0$, $y'(0) = 2$. Find first two nonzero terms of the power series of the solution.

2. [10 points] Consider $(x^2 + 4)y'' + (x^2 - 1)y' - 4x^2 y = 0$. DO NOT TRY TO SOLVE THIS. Merely find a lower bond on the radius of convergence of the series solution centered about $x = 1$.

3. [20 points]

   a) [5 points] Find the general solution to $y'' + 4y' + 3y = 0$.

   b) [10 points] Find a particular solution to $y'' + 4y' + 3y = 6\sin t + 2\cos t$.

   c) [5 points] Just suppose $y(t) = C_1 e^{-3t} + C_2 e^{-t} + \sin t - \cos t$. Find C_1 and C_2 so that $y(0) = 0$ and $y'(0) = 0$.

4. [10 points] Find the general solution to $y''' + y'' - y' - y = 0$.

5. [10 points] Which pairs of functions below are linearly independent? Justify your answer. (Recall $\cos(2x) = \cos^2(x) - \sin^2(x)$.)

   a) $\cos(t^2)$, $\sin(t^2)$

   b) $\cos^2(x)$, $1 + \cos(2x)$

6. [10 points] Find the general solution of $16y'' - 8y' + 145y = 0$.

7. [10 points] Rewrite $3\cos(2t) + 4\sin(2t)$ in the form $R\cos(\omega t - \delta)$.

8. [20 points] Consider $ty'' - y' + 4t^3 y = 0$, $t > 0$.

   (a) [5 points] Show that $y_1(t) = \sin(t^2)$ is a solution.

   (b) [8 points] Let $y_2(t) = v(t)\sin(t^2)$. Substitute y_2 into the original equation and obtain a differential equation for v. (It should involve only v'' and v'. The v terms will cancel out.)

   (c) [7 points] Find $v(t)$ and then $y_2(t)$. Hints: $\int \cot(u)\,du = \ln|\sin(u)| + C$ and $\int \csc^2(u)\,du = \cot(u) + C$. (It gets messy.)