

Name: _____ Section time: _____

ONLY NON GRAPHING CALCULATORS ALLOWED

1. [10 points] a. Solve $\frac{dy}{dt} = y(K - y)$ where $K > 0$ and $y(0) = y_0 > 0$. Find y as a function of t .
b. What is $\lim_{t \rightarrow \infty} y(t)$?

2. [10 points] Find the general solution to

$$\frac{dy}{dx} = \frac{y}{x} + \csc\left(\frac{y}{x}\right).$$

You do not need to solve for y or x but may leave your answer as a relation.

3. [10 points] a. Show that

$$(3x^3y^2 + 2x^2y) + (x^4y + 2xy^2)y' = 0$$

is not exact.

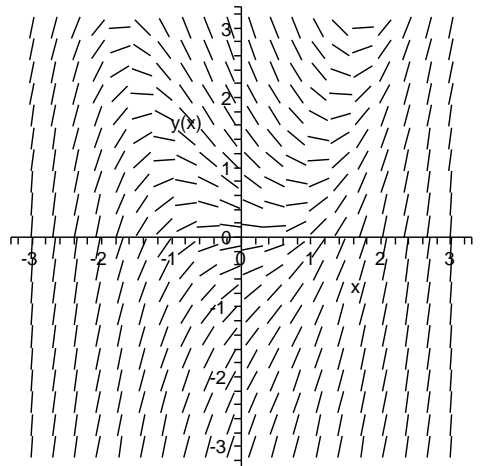
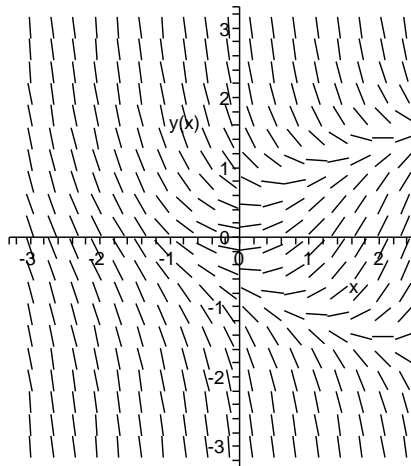
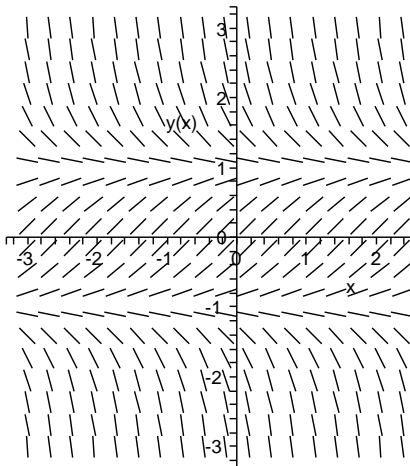
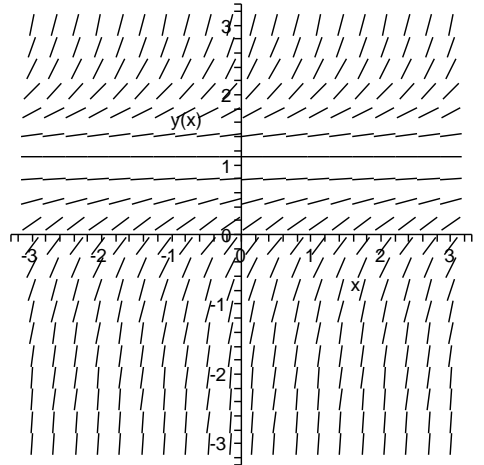
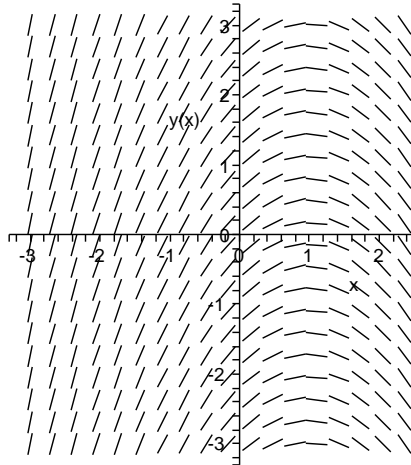
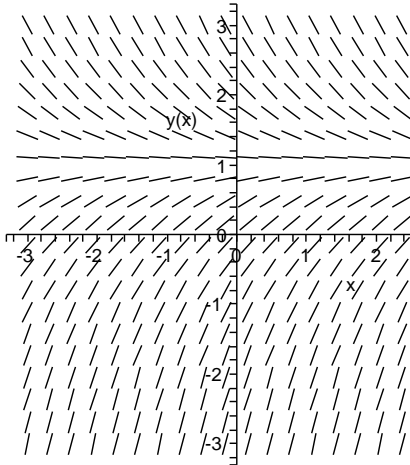
- b. Multiply through by $1/xy$ and show the new equation is exact.
c. Find the general solution. You may leave your answer as a relation.
4. [10 points] Find the general solution to

$$y' + t^2y = t^2y^4.$$

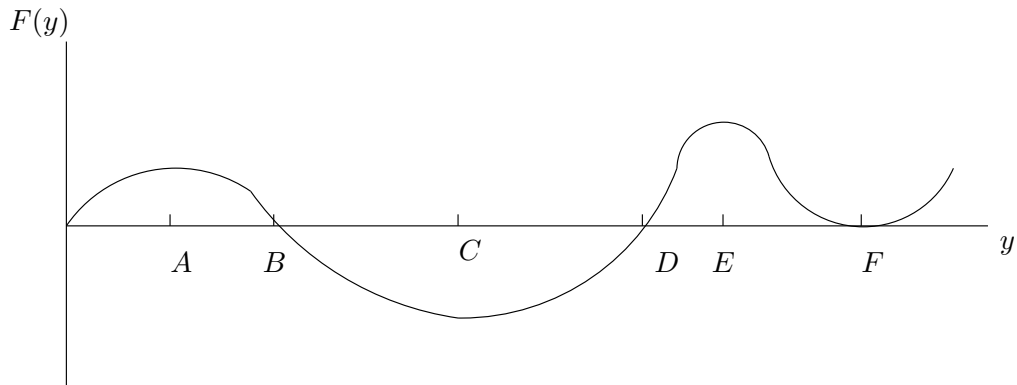
Hint: It is a Bernoulli type equation.

5. [10 points] You are in the middle of giving a presentation as part of a job interview. Your roommate, always the prankster, has mixed up your direction field slides and inserted one you weren't going to use. Match the differential equations with their corresponding direction fields. (Each correct match is worth 2 points, each incorrect match is -1 point.)

- (a) $y' = 1 - x$
(b) $y' = x - y^2$
(c) $y' = (1 - y)^2$
(d) $y' = 1 - y^2$
(e) $y' = 1 - y$



6. [10 points] Suppose $y'(t) = F(y(t))$, where the graph of $F(y)$ is given below. Carefully draw several solution curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $y(t)$ and t are non-negative.
7. [20 points] A boat weighing 480 lbs is pushed forward by a 10 lb force (from its motor). It starts from rest. The water resistance force is twice the velocity.
- Find a differential equation for the velocity of the boat.
 - Solve it expressing velocity v (ft/sec) and a function of time t seconds.



c. Find the limit of $v(t)$ as $t \rightarrow \infty$.

8. [10 BONUS points] In this problem you are to solve

$$\frac{dy}{dx} = \frac{x}{x^2y + y^3}. \quad (*)$$

None of our methods will work. Instead we let $u = x^2$ and use the fact that $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ to convert (*) into an equation involving only u and y . Then use the fact that

$$\frac{du}{dy} = \frac{1}{\frac{dy}{du}}$$

to rewrite the equation with u a function of y . It will be linear in u' and u .

You may use $\int y^3 e^{-y^2} dy = \frac{-1}{2}(1 + y^2)e^{-y^2} + C$.