

Name: \_\_\_\_\_ Section time: \_\_\_\_\_

**SCIENTIFIC CALCULATORS ALLOWED**

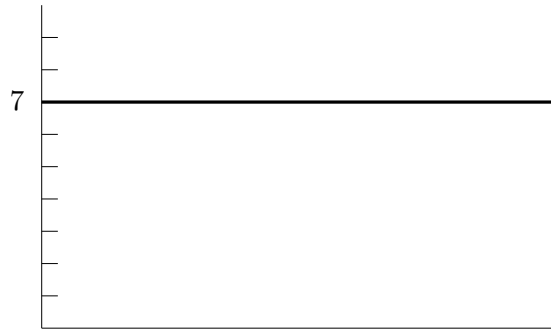
1. [20 points] Suppose  $y'' + (\tan t)y' + ty = 0$  and that  $y(0) = 2$  and  $y'(0) = 0$ .
  - a. Find the first five terms of the power series solution for  $y(t)$  centered about  $t = 0$ .
  - b. Give a lower bound for the radius of convergence.
2. [20 points] Consider  $y'' + xy' - 2y = 0$ .
  - a. Find the general power series solution including a recursive formula for  $a_n$ .
  - b. Now suppose  $y(0) = 1$  and  $y'(0) = 0$ . Find the values of all the  $a_n$ 's. (The series should terminate quickly.)
3. [20 points] Consider the wave  $f(x) = |\sin x|$ .
  - a. Graph this function for several periods. What is the period? Is it even, odd or neither?
  - b. Set up the integrals needed to find the Fourier Series of  $f(x)$ .
  - c. Find  $a_0$ ,  $a_2$  and  $b_{13}$ . (The  $a_n$ 's are the cosine coefficients and the  $b_n$ 's are the sine coefficients as in the text and in class.) Hint:  $\cos 2x = \cos^2 x - \sin^2 x$ .
4. [20 points] Find the solution to the heat conduction problem

$$u_{xx} = 2u_t, \quad 0 < x < 1, \quad t > 0;$$

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0;$$

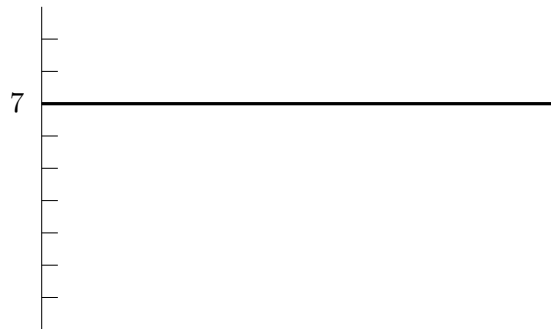
$$u(x, 0) = \sin(2\pi x) + 2 \sin(7\pi x).$$

5. [20 points] a. A one meter metal rod initially has uniform temperature of  $7 C^\circ$ . The left end is then set to  $0 C^\circ$  and the right end to  $10 C^\circ$ . As  $t \rightarrow \infty$  what will the temperature distribution approach? Graph this and sketch graphs for several intermediate temperature distributions we would expect to see.



one meter

b. A one meter metal rod initially has uniform temperature of  $7\text{ }C^\circ$ . The left end is then set to  $0\text{ }C^\circ$  while the right end is insulated. As  $t \rightarrow \infty$  what will the temperature distribution approach? Graph this and sketch graphs for several intermediate temperature distributions we would expect to see.



one meter