Spring 2017 **Final Exam** **Math 305**

Only Scientific Calculators Allowed

Print Name: _____

In problems 1-8 find the general solution. Solve for y as a function od the indepeneny variable unless indicated overwise.

1. [5 points] $ty' + 2y = 4t^2$.
2. [5 points] $\dfrac{dy}{dx} = \dfrac{4x - x^3}{4 + y^3}$ (You may leave your answer as a relation between x and y.)
3. [5 points] $\frac{dy}{dx} = \frac{y}{x} + \csc\left(\frac{y}{x}\right)$. (You may leave your answer as a relation between x and y.)
4. [5 points] $\dfrac{dy}{dx} = -\dfrac{3x^2y + xy^2}{x^3 + x^2y}$. (You may leave your answer as a relation between x and y.)
5. [10 points] $y' + t^2y = t^2y^4$.
6. [10 points] $y'' + \dfrac{y'}{t+1} = 2$.
7. [10 points] $y'' - 4y' + 4y = e^{2x}$
8. [10 points] $y''' - y'' + 2y' - 2y = 0$
9. [20 points] Show that $y_1 = x$ is a solution of

$$x^2y'' - x(x+2)y' + (x+2)y = 0.$$

   To find a second linearly indepenent solution let $y_2 = v(x)y_1(x) = v \cdot x$. Substitute this into the differential equation to get a differential equation in v. Solve it and then solve for y_2. This is the *Reduction of Order Method*.

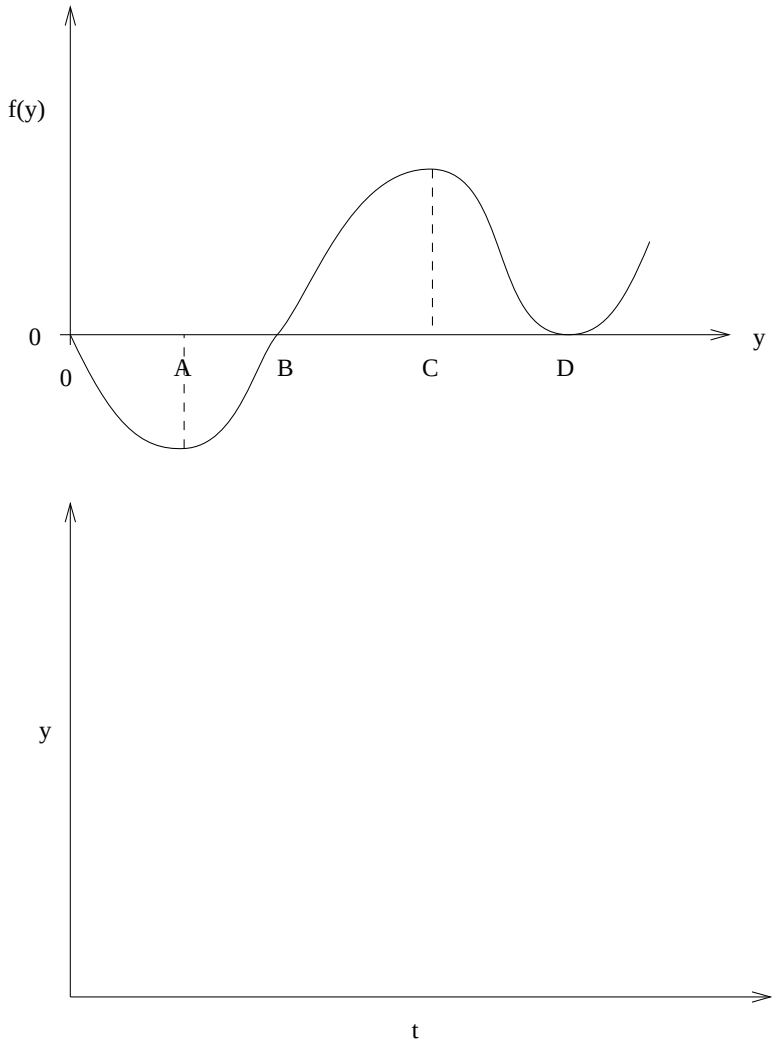
10. [20 points] A body of mass m falls from rest in a medium offering resistance proportional to the square of the velocity, v^2. Find the relation between the velocity v and the time t. Find the limiting velocity as $t \to \infty$.

    Hint:

$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{1}{2ab}\ln\left|\frac{a+bx}{a-bx}\right| + C,$$

    where a and b are positive constants.

11. [20 points] Suppose $N'(t) = F(N(t))$, where the graph of $F(N)$ is given below. Carefully draw the integral curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $N(t)$ and t are non-negative.



12. [20 points] A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ grams/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount if salt in the tank as $t \to \infty$.

13. [20 points] Consider the initial value problem $y'' + (\ln x)y' + xy = 0$, $y(1) = 1$, $y'(1) = 0$. Find the first five terms of the series solution centered about $x = 1$.

14. [20 points] Let

$$f(x) = \begin{cases} 1 & x \in [0,1] \\ x & x \in [1,2] \\ 2 & x \in [2,3] \end{cases} .$$

Graph the even and odd extensions of $f(x)$. Find the first two terms in of the Fourier series of the even periodic extension.

Hints:

$$\int x\cos(ax)\,dx = \frac{\cos(ax) + ax\sin(ax)}{a^2} + C$$

$$\int x\sin(ax)\,dx = \frac{\sin(ax) - ax\cos(ax)}{a^2} + C$$