Math 305 Test 2 Spring 2017

Name: _____ Section time: _____

ONLY NON GRAPHING CALCULATORS ALLOWED

1. [15 points] Find the general solution to $y'' + 3y' - 4y = \sin x$.

2. [10 points] Find the general solution to $y''' - y'' + 2y' - 2y = 0$.

3. [15 points] Find the general solution to $y'' - 4y' + 4y = e^{2x}$

4. [5 points] Suppose we given $y'' + p(t)y' + q(t)y = 0$, with $p(t)$ and $q(t)$ continuous everywhere, and are told that $t(t+1)(t-1)$ and $t^2 + 1$ are solutions. Is this possible? Explain. Hint: graph them.

5. [10 points] Find the first five terms of the Taylor polynomial of the solution to

$$y'' + xy' - 2y = 0, \quad y(0) = 1, y'(0) = 0.$$

6. [5 points] Convert $3\cos 2t + 4\sin 2t$ into the form $R\cos(w_o t - \delta)$. (Find δ in radians.)

7. [5 points] Find the general solution to $y'' + y = 0$. (This is a freebie.)

8. [5 points] Suppose we are given $y'' + y = t(1 + \sin t)$. To apply the *Method of Undetermined Coefficients* what should the form of the particular solution be?

9. [15 points] Show that $y_1 = x$ is a solution of

$$x^2y'' - x(x+2)y' + (x+2)y = 0.$$

   To find a second linearly indepenent solution let $y_2 = v(x)y_1 = vx$. Substitute this into the differential equation to get a differential equation in v. Solve it. This is the *Reduction of Order Method*.

10. [15 points] A 4 kilogram object is attached to the lower end of a spring whose upper end is attached to the ceiling. The spring constant k is 2 kg/meter. The resistance to the motion is $\gamma = 2$ N-s/m times the velocity of the object. The object is set in motion by pushing it up 0.5 m and then letting it go.

    Set up and solve a differential equation to model this mass-spring system. Is this system an exmaple of small-damping, over-damping or critical-damping?