Math 305 Test 3 Spring 2017

Name: _____ Section time: _____

Only Scientific Calculators are Allowed

1. [15 points] Suppose $y'' - \frac{1}{t-3}y' - 2ty = 0$ and that $y(0) = 2$ and $y'(0) = 1$.

   a. Find the first five terms of the power series solution for $y(t)$ centered about $t = 0$.

   b. Give a lower bound for the radius of convergence.

2. [15 points] Consider $y'' + xy' - 2y = 0$.

   a. Find the general power series solution centered about zero. Include a recursive formula for a_n.

   b. Now suppose $y(0) = 1$ and $y'(0) = 0$. Find the values of all the a_n's. (The series should terminate quickly.)

3. [10 points] Consider the partial differential equation below.

$$u_{xxt} - u_{xt} = u_x,$$

   where u is a function of independent variables x and t. Suppose $u(x,t) = X(x)T(t)$. Derive the ordinary differential equations below.

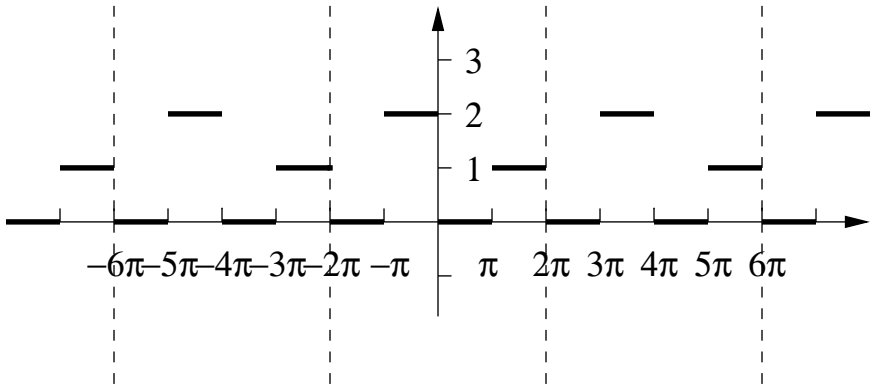
$$X'' + (1-a)X' = 0 \quad \& \quad aT' - T = 0.$$

4. [10 points] Find all positive values of γ such that the boundary value problem

$$y''(t) + \gamma^2 y(t) = 0, \;\; y(0) = y(2) = 0,$$

   has nontrivial solutions.

5. [20 points] Consider the periodic function $f(x)$ in the graph below.



We wish to calculate a few terms of its Fourier series:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right).$$

o. What is L?

i. What is a_0? (Be careful.)

ii. What is a_2?

iii. What is b_2?

6. [4 points] Suppose $f(x)$ is an even function and $g(x)$ is an odd function. Prove that $h(x) = f(f(x)) + g(x)g(x)$ is even.

7. [6 points] Let $f(x)$ be any function. Define

$$g(x) = \frac{f(x) - f(-x)}{2} \quad \& \quad h(x) = \frac{f(x) + f(-x)}{2}.$$

Show the following.

a. $g(x) + h(x) = f(x)$.

b. $g(x)$ is odd.

c. $h(x)$ is even.

Thus, g and h are called, respectively, the odd and even parts of f.

8. [10 points] Find the solution to the heat conduction problem

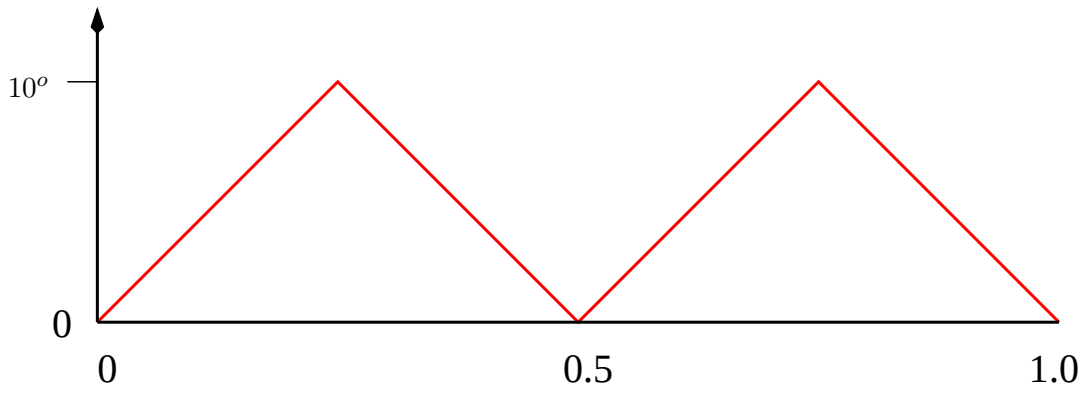
$$u_{xx} = 16u_t, \quad 0 < x < 1, \quad t > 0;$$

$$u(0,t) = 0, \quad u(1,t) = 0, \quad t > 0;$$

$$u(x,0) = \sin(3\pi x) + 19\sin(6454\pi x).$$

9. [10 points]

a. A one meter metal rod initially has the temperature distribution shown below. Suppose the ends are held at 0 degrees. As $t \to \infty$ what will the temperature distribution approach? Graph this and sketch graphs for several intermediate temperature distributions we would expect to see.



b. A one meter metal rod initially has the temperature distribution shown below. Suppose the ends are insulated. As $t \to \infty$ what will the temperature distribution approach? Graph this and sketch graphs for several intermediate temperature distributions we would expect to see.