

Example 6 from § 23.

$$\sum_{n=0}^{\infty} 2^{-n} X^{3n} = 1 + \frac{1}{2} X^3 + \frac{1}{4} X^6 + \frac{1}{8} X^9 + \dots$$

$$= 1 + 0X + 0X^2 + \frac{1}{2}X^3 + 0X^4 + 0X^5 + \frac{1}{4}X^6 + 0X^7 + 0X^8 + \frac{1}{8}X^9 + \dots$$

Thus $\lim \sqrt[n]{|a_n|}$ does not exist.

$$\liminf \sqrt[n]{|a_n|} = 0$$

$$\limsup \sqrt[n]{|a_n|} = ? \quad a_n = \begin{cases} 0 & n \bmod 3 = 1 \\ 0 & n \bmod 3 = 2 \\ 2^{-\frac{n}{3}} & n \bmod 3 = 0 \end{cases}$$

$$\limsup \sqrt[n]{2^{-n/3}} = \limsup 2^{-1/3} = 2^{-1/3}.$$

$$\text{Thus } R = 2^{1/3} \approx 1.25992$$