Geometric Series Examples

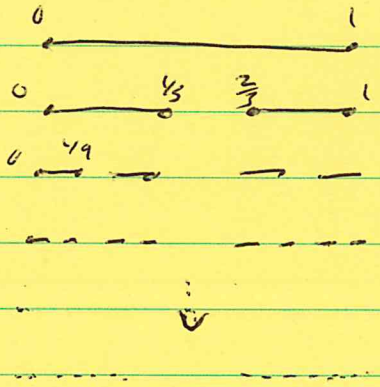
1. $\sum_{k=0}^{\infty} \left(\frac{4}{5}\right)^k = \frac{1}{1-\frac{4}{5}} = \frac{1}{\frac{1}{5}} = 5.$

2. $\sum_{k=0}^{\infty} \left(\frac{-1}{3}\right)^k = \frac{1}{1-\left(\frac{-1}{3}\right)} = \frac{1}{\frac{4}{3}} = \frac{3}{4}.$

3. $\frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \cdots = ?$

$$\sum_{k=1}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^k = \frac{1}{3}\left[\frac{1}{1-\frac{2}{3}} - 1\right] = \frac{1}{3}\left[3-1\right] = \frac{2}{3}.$$

4. How long is the middle thirds Cantor set?



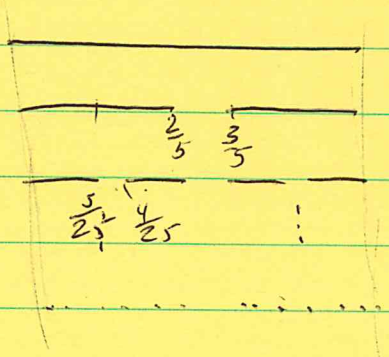
See pages 88-90 in Ross textbook.

← Cantor set.

$$\text{Length} = 1 - \left(\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \cdots\right)$$

$$= 1 - \sum_{k=0}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^k = 1 - \frac{1}{3}\left[\frac{1}{1-\frac{2}{3}}\right] = 1 - \frac{3}{3} = 0.$$

5. What if we cut out the middle $\frac{1}{5}$ each time?



$$L = 1 - \left(\frac{1}{5} + \frac{2}{25} + \frac{4}{125} + \frac{8}{625} + \cdots\right)$$

$$= 1 - \sum_{k=0}^{\infty} \frac{1}{5}\left(\frac{2}{5}\right)^k = 1 - \frac{1}{5}\left(\frac{1}{1-\frac{2}{5}}\right) = 1 - \frac{1}{5}\cdot\left(\frac{5}{3}\right) = \frac{2}{3}.$$

Called a <u>thick Cantor set</u>.

6. Look up Koch snowflake.