Equivalent Limit Definitions

These are formal definitions of each type of limit for functions of real numbers using sequences. In each case it is assumed no term of the sequence (a_n) is equal to a and all terms are in the domain of the function. We use $a_n \to a^+$ to mean $a_n \to a$ and all $a_n > a$; we use $a_n \to a^-$ to mean $a_n \to a$ and all $a_n < a$. Assume L and a are real numbers.

1. $\lim_{x\to a} f(x) = L$ means $a_n \to a \implies f(a_n) \to L$. It is assumed the domain of f contains a set of the form $(a-\mu, a) \cup (a, a+\mu)$ for some $\mu > 0$.
    - As x approaches a from either side $f(x)$ approaches L.
2. $\lim_{x\to a^+} f(x) = L$ means $a_n \to a^+ \implies f(a_n) \to L$. It is assumed the domain of f contains a set of the form $(a, a+\mu)$ for some $\mu > 0$.
    - As x approaches a from the right side $f(x)$ approaches L.
3. $\lim_{x\to a^-} f(x) = L$ means $a_n \to a^- \implies f(a_n) \to L$. It is assumed the domain of f contains a set of the form $(a-\mu, a)$ for some $\mu > 0$.
    - As x approaches a from the left side $f(x)$ approaches L.
4. $\lim_{x\to \infty} f(x) = L$ means $a_n \to \infty \implies f(a_n) \to L$. It is assumed the domain of f contains a set of the form (b, ∞) for some $b \in \mathbb{R}$.
    - As x grows positively without bound $f(x)$ approaches L.
5. $\lim_{x\to -\infty} f(x) = L$ means $a_n \to -\infty \implies f(a_n) \to L$. It is assumed the domain of f contains a set of the form $(-\infty, b)$ for some $b \in \mathbb{R}$.
    - As x grows negatively without bound $f(x)$ approaches L.
6. $\lim_{x\to a} f(x) = \infty$ means $a_n \to a \implies f(a_n) \to \infty$. It is assumed the domain of f contains a set of the form $(a-\mu, a) \cup (a, a+\mu)$ for some $\mu > 0$.
    - As x approaches a from either side $f(x)$ grows positively without bound.
7. $\lim_{x\to a^+} f(x) = \infty$ means $a_n \to a^+ \implies f(a_n) \to \infty$. It is assumed the domain of f contains a set of the form $(a, a+\mu)$ for some $\mu > 0$.
    - As x approaches a from the right side $f(x)$ grows positively without bound.
8. $\lim_{x\to a^-} f(x) = \infty$ means $a_n \to a^- \implies f(a_n) \to \infty$. It is assumed the domain of f contains a set of the form $(a-\mu, a)$ for some $\mu > 0$.
    - As x approaches a from the left side $f(x)$ grows positively without bound.
9. $\lim_{x\to a} f(x) = -\infty$ means $a_n \to a \implies f(a_n) \to -\infty$. It is assumed the domain of f contains a set of the form $(a-\mu, a) \cup (a, a+\mu)$ for some $\mu > 0$.
    - As x approaches a from either side $f(x)$ grows negatively without bound.
10. $\lim_{x\to a^+} f(x) = -\infty$ means $a_n \to a^+ \implies f(a_n) \to -\infty$. It is assumed the domain of f contains a set of the form $(a, a+\mu)$ for some $\mu > 0$.
    - As x approaches a from the right side $f(x)$ grows negatively without bound.
11. $\lim_{x\to a^-} f(x) = -\infty$ means $a_n \to a^- \implies f(a_n) \to -\infty$. It is assumed the domain of f contains a set of the form $(a-\mu, a)$ for some $\mu > 0$.
    - As x approaches a from the left side $f(x)$ grows negatively without bound.
12. $\lim_{x\to \infty} f(x) = \infty$ means $a_n \to \infty \implies f(a_n) \to \infty$. It is assumed the domain of f contains a set of the form (b, ∞) for some $b \in \mathbb{R}$.
    - As x grows positively without bound $f(x)$ grows positively without bound.
13. $\lim_{x\to \infty} f(x) = -\infty$ means $a_n \to \infty \implies f(a_n) \to -\infty$. It is assumed the domain of f contains a set of the form (b, ∞) for some $b \in \mathbb{R}$.
    - As x grows positively without bound $f(x)$ grows negatively without bound.
14. $\lim_{x\to -\infty} f(x) = \infty$ means $a_n \to -\infty \implies f(a_n) \to \infty$. It is assumed the domain of f contains a set of the form $(-\infty, b)$ for some $b \in \mathbb{R}$.
    - As x grows negatively without bound $f(x)$ grows positively without bound.
15. $\lim_{x\to -\infty} f(x) = -\infty$ means $a_n \to -\infty \implies f(a_n) \to -\infty$. It is assumed the domain of f contains a set of the form $(-\infty, b)$ for some $b \in \mathbb{R}$.
    - As x grows negatively without bound $f(x)$ grows negatively without bound.