

Examples

$$1. \limsup \left\{ \frac{1}{n} \right\} = \lim_{N \rightarrow \infty} \sup \left\{ \frac{1}{n}, n > N \right\}$$

$$\text{Let } a_N = \sup \left\{ \frac{1}{n}, n > N \right\}.$$

$$a_1 = \frac{1}{2}$$

$$a_2 = \frac{1}{3}$$

$$a_3 = \frac{1}{4}$$

$$a_N = \frac{1}{N+1}$$

$$\text{Hence } \limsup \left\{ \frac{1}{n} \right\} = \lim_{N \rightarrow \infty} \frac{1}{N+1} = 0.$$

$$2. \liminf \left(\frac{1}{n} \right) = \lim_{N \rightarrow \infty} \inf \left\{ \frac{1}{n}, n > N \right\}$$

$$\text{Let } b_N = \inf \left\{ \frac{1}{n}, n > N \right\}.$$

$$b_1 = 0, b_2 = 0, \dots \quad \text{Hence } \liminf \left(\frac{1}{n} \right) = 0.$$

$$3. \limsup \left((-1)^n \right) = \lim_{N \rightarrow \infty} \sup \left\{ (-1)^n \mid n > N \right\} = \lim_{N \rightarrow \infty} \sup \{-1, 1\} = 1$$

$$4. \liminf \left((-1)^n \right) = -1.$$

$$5. \limsup \left((-1)^n + \frac{1}{n} \right) = \lim_{N \rightarrow \infty} \sup \left\{ (-1)^n + \frac{1}{n} \mid n > N \right\}$$

$$\text{Let } a_N = \sup \left\{ (-1)^n + \frac{1}{n}, n > N \right\}.$$

$$a_1 = \sup \left\{ \frac{1}{2}, -\frac{2}{2}, \frac{1}{4}, -\frac{1}{5}, \dots \right\} = \frac{1}{2}$$

$$a_2 = \frac{1}{4}$$

$$a_3 = \frac{1}{4}$$

$$a_4 = \frac{1}{6}$$

$$a_N \rightarrow 1.$$

$$6. \liminf \left((-1)^n + \frac{1}{n} \right) = -1$$