

Hwk Set 2. Due Wednesday, January 22. Proofs are to be written in grammatically correct English sentences.

1. Prove that $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all natural numbers n .

2. Prove that $\sum_{i=1}^n (2i - 1) = n^2$ for all natural numbers n .

3. Prove $n^3 + 5n + 6$ is divisible by 3 for all natural numbers n .

4. Assume $r \neq 1$. Prove that

$$\sum_{k=0}^{n-1} r^k = \frac{r^n - 1}{r - 1},$$

for all natural numbers n .

5. Show that $3^{\frac{2}{7}}$ is not rational.

6. Show that $(2 + \sqrt{7})^{\frac{1}{5}}$ is not rational.

7. Graph $y = ||3x + 5| - 2|$.

8. Prove that \mathbb{Q} is countable.

9. Prove that \mathbb{R} is not countable.

10. (Bonus!) Show that $\log_2 3$ is not rational.

11. (Bonus!) Prove that there are infinitely many prime natural numbers. Hint: Suppose the only prime numbers are p_1, p_2, \dots, p_n . Think about $q = p_1 p_2 \cdots p_n + 1$.