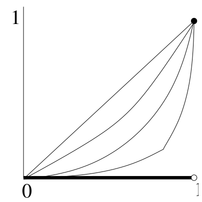
Introduction

Why do you have to take this course?

1. In 1734 the English philosopher George Berkeley (pronounced *Barclay*) wrote a book entitled *The Analyst: A Discourse Addressing an Infidel Mathematician.* The mathematician in question was Isaac Newton. Berkeley's book excoriates Newton's calculus as illogical nonsense because it lacked a sound theoretical basis. (See the Optional Reading link.)

2. Problems with Convergence. In about 1820 Cauchy "proved" that if $f_n(x) \to f(x)$ for all x and if the f_n's are continuous then the limit function f must also be continuous. But this is false!

Let $f_n(x) = x^n$ on the set [0,1]. The limit is ...



This caused big headaches in an area of applied mathematics called *Fourier series.* It took mathematicians about 50 years to figure out what the problem was and that an additional hypothesis was needed for Cauchy's result to hold. (This is called *uniform convergence* which we will study later.) See *Proofs and Refutations,* by Imre Lakatos, 1967, Appendix 1.

Thus, mathematicians were forced to develop a sound theoretical basis for Newton's calculus for both philosophical and practical reasons.

3. Another important reason this course required is because many of you will be teaching calculus or precalculus in high schools. We normally gloss over the theory when teaching calculus to newbies. But it still comes up occasionally and some students will ask about how we know certain results hold up. So, the instructor needs to have a sound understanding of theory and be able to show students selected bits of it as appropriate.

You should read Sections 1-3 of the textbook before the next lecture.