Math 405 Practice Test 1.2 Spring 2002

Part I: In class, no calculators.

1. [20 points] Find the solution to the initial value problem $\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \mathbf{x}$ with $\mathbf{x}(0) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$.

2. [20 points] Find the general solution of $\mathbf{x}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & -1 & 3 \end{bmatrix} \mathbf{x}$.

3. [20 points] Find the general solution of $\mathbf{x}' = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix}$.

4. [20 points] Suppose that, for a given matrix $\mathbf{A}$, there is a nonzero vector $\mathbf{x}$ such that $\mathbf{A}\mathbf{x} = \mathbf{0}$. Show that there is also a nonzero vector $\mathbf{y}$ such that $\mathbf{A}^* \mathbf{y} = \mathbf{0}$.

5. [20 points] Prove that $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{A}\mathbf{B} + \mathbf{A}\mathbf{C}$, for matrices.

   Part II: In computer lab.

6. [10 points] Transform $\mathbf{x}''' = \mathbf{x}$ into a system of first order equations. Find the general solution with the computer.

7. [10 points] Do a phase plot for $\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \mathbf{x}$. and overlay plots of solution curves for $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$.