No notes, books, or calculators are allowed.

1. Find the eigenvalues and eigenvectors.

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 4 & -4 & -6 \\ 4 & 14 & 12 \\ -3 & -6 & -3 \end{bmatrix} \qquad \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}$$

2. Determine if each set of vector valued functions is linearly dependent or linearly independent.

$$\left\{ \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}, \begin{bmatrix} \cos t - \sin t \\ \cos t \end{bmatrix} \right\} \qquad \left\{ \begin{bmatrix} e^t \\ e^{2t} \\ e^{3t} \end{bmatrix}, \begin{bmatrix} 2e^t \\ e^{2t} \\ -e^{3t} \end{bmatrix}, \begin{bmatrix} 3e^t \\ 2e^{2t} \\ 0 \end{bmatrix} \right\}$$

3. Find the general solution to $\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 15 & 39 \\ -6 & -15 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$. Express your answer in terms of real valued functions. Sketch a rough phaseportrait. Hint: The eigenvalues are $\pm 3i$ with coresponding eigenvectors $\begin{bmatrix} -13 \\ 5 \mp i \end{bmatrix}$.

4. Find the general solution to $\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$. Hint: It has repeated eigenvalue 2. You will need to find an eigenvector and a generalized eigenvector.

5. Find the general solution to $\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$. Find the solution for initial values $x(0) = 2$ and $y(0) = 0$. Sketch a rough phaseportrait; include the eigenspaces and several solution curves. Hint: the eigenvalues are -1 and -3, with respective eiganvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

6. Find the general solution to $\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 2 \\ \sin t \end{bmatrix}$. Hint: The eigenvalues are 1 and 2 with corresponding eigenvectos $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.