Math 405 Practice Test 1.1 Spring 2002

Part 1: In class, no calculators.

1. [20 points] Find the general solution to $\mathbf{x}' = \begin{bmatrix} -3 & \frac{5}{2} \\ -\frac{5}{2} & 2 \end{bmatrix} \mathbf{x}$.

2. [20 points] Solve the initial value problem $\mathbf{x}' = \begin{bmatrix} 0 & 0 & -1 \\ 2 & 0 & 0 \\ -1 & 2 & 4 \end{bmatrix} \mathbf{x}$, with $\mathbf{x}(0) = \begin{bmatrix} 7 \\ 5 \\ 5 \end{bmatrix}$.

   What happens as $t \to \infty$?

3. [20 points] Find the general solution to $\mathbf{x}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix}$.

4. [20 points] Show that if $\mathbf{A}$ is a diagonal matrix with diagonal elements a_1, a_2, ..., a_n, then $\exp(\mathbf{A}t)$ is also a diagonal matrix with elements $\exp(a_1 t)$, $\exp(a_2 t)$, ..., $\exp(a_n t)$.

5. [20 points] Assume that $a + d < 0$ and $ad - bc > 0$. Show that all solutions of $\mathbf{x}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ approach zero as $t \to \infty$.

   Part II: In computer lab.

6. [10 points] Do a phase plot for $\mathbf{x}' = \begin{bmatrix} -3 & \frac{5}{2} \\ -\frac{5}{2} & 2 \end{bmatrix} \mathbf{x}$ and overlay plots of solution curves for $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$.

7. [10 points] Find the solutions to $\mathbf{x}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix}$, for $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$.