

Part I: In class, no calculators.

1. [20 points] Consider the nonlinear system $x' = 1 + 2y$ and $y' = 1 - 3x^2$. Find and classify the critical points. Sketch a phase portrait.
2. [20 points] Consider the nonlinear system $x' = -x - xy^2$ and $y' = -y - x^2y$. Show that the critical point $(0,0)$ is asymptotically stable by finding a suitable Liapunov function.
3. [20 points] Consider the nonlinear system $x' = x + y + x^3 - y^2$ and $y' = -x + 2y + x^2y + y^3/3$. Find and classify the limit cycles, or show that there are none.
4. [20 points] Consider the nonlinear system $x' = x + y - x(x^2 + y^2)$ and $y' = -x + y - y(x^2 + y^2)$. Find and classify the limit cycles, or show that there are none.
5. [20 points] Assuming that the trajectory corresponding to a solution of an autonomous system is closed, show that it is periodic.

Part II: In computer lab.

6. [20 points] Produce phase portraits for problems 1 and 4.