Math 405 Practice Test 2.2 Spring 2002

Part I: In class, no calculators.

1. [20 points] Consider the nonlinear system $x' = 8 + y^3$ and $y' = 4 - x^2$. Find and classify the critical points. Sketch a phase portrait.

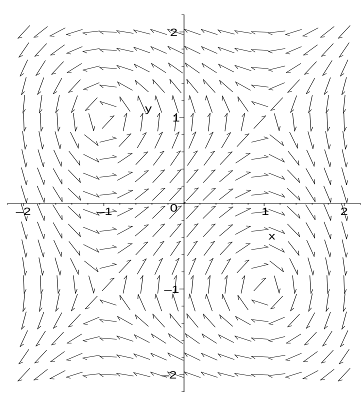
2. [20 points] Consider the nonlinear system $x' = x - x^2 - xy$ and $y' = 3y - xy - 2y^2$. Find and classify the critical points.

3. [20 points] Explain how the predator-prey equations are derived. What are the three underlining assumptions?

4. [20 points] Use the Liaponov function $V(x, y, z) = rx^2 + \sigma y^2 + \sigma(z - 2r)^2 = c > 0$ to show that the origin is a globally asymptotically stable critical point for the Lorenz equations (see below) if $r > 1$.

$$\begin{aligned} dx/dt &= \sigma(-x + y), \\ dy/dt &= rx - y - xz, \\ dz/dt &= -bz + xy. \end{aligned}$$

5. [20 points] Find a system of equations whose phase portriat would look like the one below. Justify your answer.



Part II: In computer lab.

6. [20 points] Produce phase portraits for problems 1 and 2.