Math 452
Homework Set 1

This homework set is meant to be a review. All answers are to be written out in complete sentences with correct grammar and punctuation. It is due on the first day of class.

(1) Prove that $\sqrt{3}$ is not a rational number.
(2) Give a formal ϵ-δ style proof that $\lim_{x\to 2} 3x + 5 = 11$.
(3) Give a formal ϵ-δ style proof that $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist.
(4) Define $f(x)$ to be $x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and 0 for $x = 0$. Is f differentiable at $x = 0$? Prove your claim.
(5) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. Prove that if f has a local maximum or minimum at c then $f'(c) = 0$.
(6) Use the Intermediate Value Theorem, the Mean Value Theorem, and other facts, to prove that the function
$$f(x) = 4x^5 + x^3 + 2x + 1$$
has one and only one real zero.
(7) Prove that the sum of the first n positive odd integers is equal to n^2.
(8) Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i)^2 \, \Delta x$, where $x_i = 1 + i/n$ and $\Delta x = 1/n$. Do this directly without using integration.
(9) Express $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i^3 + x_i \sin(x_i))\Delta x$, where $x_i = i\pi/n$ and $\Delta x = \pi/n$, as a definite integral, then evaluate it.
(10) Prove that $\sum_{n=0}^{\infty} ar^n = \dfrac{a}{1-r}$ when $|r| < 1$.
(11) Find the intervals of convergence for the two series below.
a. $\sum_{n=1}^{\infty} \dfrac{x^n}{5^n n^5}$ b. $\sum_{n=1}^{\infty} \dfrac{(-2)^n x^n}{\sqrt[4]{n}}$
(12) Evaluate $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$. Hint: Taylor series.