452 Test One Overview and Review

The midterm exam will cover Chapters 1 and 2. The Cantor Set material will not be included. Chapter 1 includes the sections I told you to read on your own. Chapters 3 and 4 will be on the compressible final. The midterm will be in there parts.

Part I ($\sim$10%) is just stating definitions, facts and named theorems. Examples: Dedekind cuts, lub, glb, sup, inf, lim sup, lim inf, least upper bound property, carnality, metric space, completeness, sequential and covering compactness, connectedness, path connected, open, closed, continuity, convergence, Cauchy sequence, uniform continuity, limit points, interior, closure, boundary, dense, Cauchy-Schwartz inequality, Triangle inequality, parallelogram law, Lebesgue number lemma, Bolzano-Weierstrass theorem, Heine-Borel theorem.

Part II ($\sim$60%) will be short proofs. I'll probably list several and ask you to prove three or four of them. Mostly they will be results you have seen, but they might include similar results or minor variations. Examples follow. Prove the ϵ-principle. Prove the Triangle inequality for $\mathbb{R}^n$. Prove that an infinite sequence in $\mathbb{R}$ converges if and if it is Cauchy. Prove that lim sup of a bounded sequence exists as a real number. In a metric space prove that limits are unique when they exist and that any subsequence of a convergent sequence has the same limit as the sequence. Prove $f : M \to N$ is continuous iff $p_n \to p$ in M implies $f(p_n) \to f(p)$ in N. Prove $f : M \to N$ is continuous iff the inverse image of closed (open) sets are closed (open). Prove the continuous image of a compact set is compact. Prove that the continuous image of a connected set is connected. Prove basic facts about open and closed sets and interiors and closures. Prove that a closed subset of a compact set is compact. Prove basic facts about connected sets including the Generalized Intermediate Value theorem and that $\mathbb{R}$ is connected. The closure of a connected set is connected. Path connected sets are connected.

Part III ($\sim$30%) will be a couple of longer proofs of major results. I'll probably list several and ask you to do two. Some possibilities include the following. Prove $\mathbb{R}$ is uncountable using Cantor's diagonalization argument. The Cauchy-Schwartz inequality. A nested sequence of nonempty compacts sets is nonempty. A continuous function on a compact set is uniformly compact. Covering compactness implies sequencial compactness. The square root of 2 exists as a Dedekind cut. $\mathbb{R}$ has the least upper bound property.