Ch 4 Sec 2

Power Series

Recall the radius of convergence of a power series $\sum c_k x^k$ is

$$R = \frac{1}{\limsup_{k \to \infty} \sqrt[k]{|c_k|}}.$$

Thm If $0 < r < R$ then $\sum_{k=0}^{n} c_k x^k \rightrightarrows \sum_{k=0}^{\infty} c_k x^k$ on $[-r, r]$.

Pf Choose $\beta \in (r, R)$. $\exists N$ s.t. $k \geq N \Rightarrow |c_k|^{\frac{1}{k}} < \frac{1}{\beta}$.

If $|x| \leq r$, then $|c_k x^k| \leq \left(\frac{r}{\beta}\right)^k$. But

$\sum \left(\frac{r}{\beta}\right)^k$ is a conv. geom. series. By the

Weierstrass M-test $\sum c_k x^k$ conv. unif. on $[-r, r]$.

Thm Power series can be integrated and differentiate term by term on $(-R, R)$.

Pf See textbook, Thm 12, pgs 221-222.

Thm $C^\omega \subset C^\infty$.

Pf See textbook, Thm 13, pg 222.