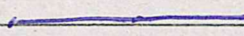
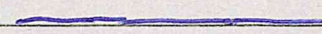
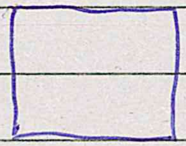
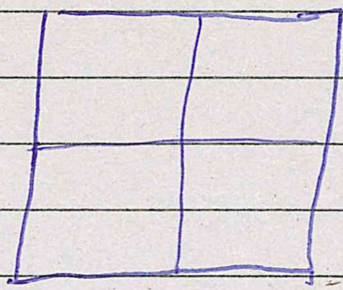
Fractals!

1-d  magnify by 2  by 3 

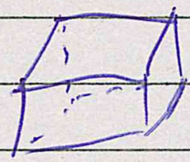
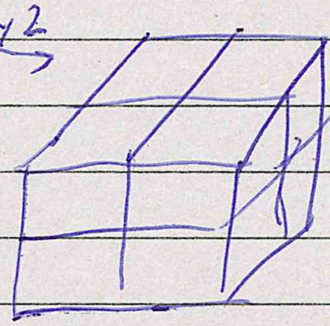
0 1

get 2 copies. get 3 copies.

2-d  $\times 2 \rightarrow$  get 4 copies.

Mag. by 3, get 9 copies.

Etc.

3-d  $\xrightarrow{\text{by } 2}$  get 8 copies.

Mag. by 3 get 27 copies.

In general, if m = magnification factor

c = # of copies, and

d = dimension,

we have $c = m^d$. Check this with the examples we did.

What happens if we ~~magnify mag~~ magnify C by 3?

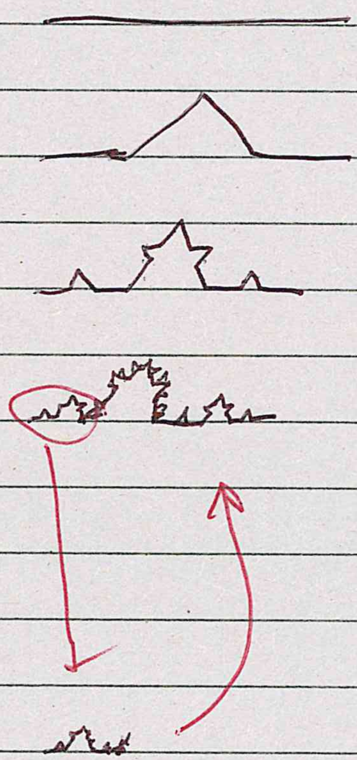
0 $\frac{1}{3}$ $\frac{2}{3}$ 1 $\longrightarrow$ 0 1 2 3

we get 2 copies of C. Thus $2 = 3^d$.

Hence $d = \log_3 2 = \frac{\ln 2}{\ln 3} \approx 0.63929754$.

The formal term for this notion of dimension is Hausdorff dimension. You can look it up. I'll just do a couple more examples. Any time $d \neq$ an integer the object is called a fractal.

Ex



Pop up the middle third.
All four segments have same length.

Repeat on each segment. All 16 segments have the sum length.
Continue.

Do this infinitely often (it is a kind of limit).

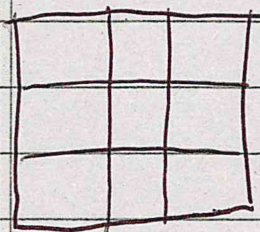
Take this piece. Magnify by 3. You get the original. It has 4 copies of the piece we magnified. Thus,

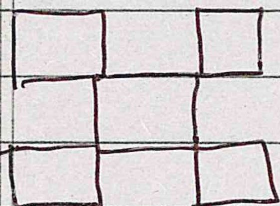
$$4 = 3^d$$

$$d = \frac{\ln 4}{\ln 3} \approx 1.098612289\ldots$$

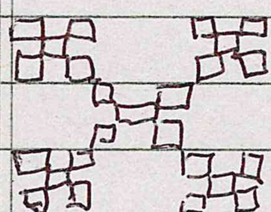
Look up Koch snowflake.

Start with a square. Divide it into 9 equal squares as shown. Remove the 4 not in a corner but one along an edge.

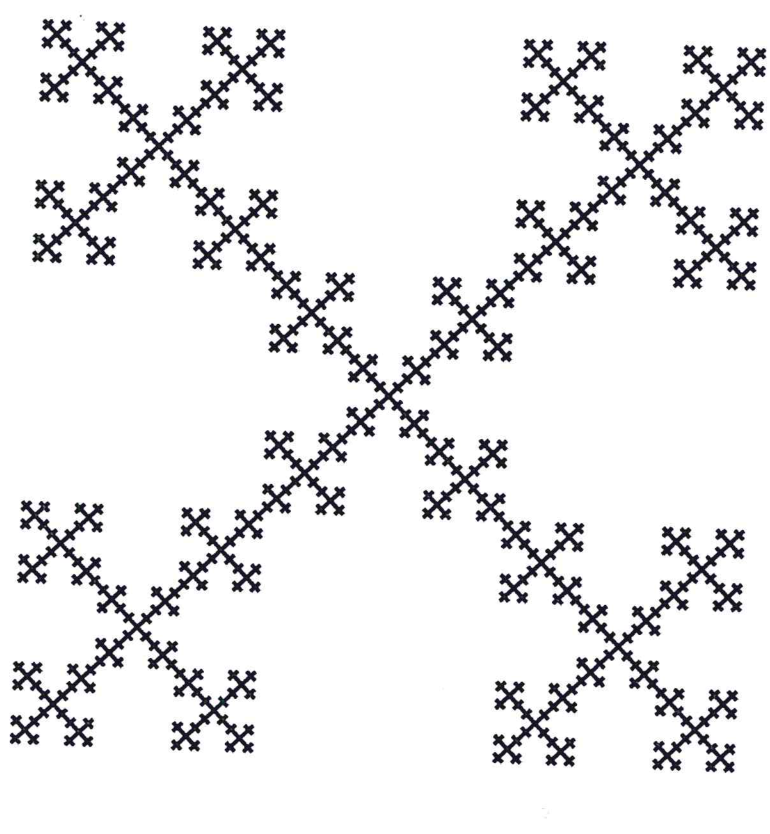




Repeat this inside each of the five remaining squares.



Repeat this inside each of the 25 remaining squares.



etc, etc...

The final result is called the Vicsek fractal. The dim. is

$$\frac{\ln 5}{\ln 3} = 1.46497352\ldots$$

From Wikipedia

Some References

Devaney, R. L. *Complex Dynamical Systems: The Mathematics Behind the Mandelbrot and Julia Sets.* Providence, RI: Amer. Math. Soc., 1994.

Devaney, R. L. and Keen, L. *Chaos and Fractals: The Mathematics Behind the Computer Graphics.* Providence, RI: Amer. Math. Soc., 1989.

Falconer, K. J. *The Geometry of Fractal Sets, 1st pbk. ed., with corr.* Cambridge, England: Cambridge University Press, 1986.

Mandelbrot, B. B. *Fractals: Form, Chance, & Dimension.* San Francisco, CA: W. H. Freeman, 1977.

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