

Math 452
Homework Set 1

This homework set is meant to be a review. All answers are to be written out in complete sentences with correct grammar and punctuation. It is due on the first day of class.

- (1) Prove that $\sqrt{3}$ is not a rational number.
- (2) Give a formal ϵ - δ style proof that $\lim_{x \rightarrow 2} 3x + 5 = 11$.
- (3) Give a formal ϵ - δ style proof that $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$ does not exist.
- (4) Define $f(x)$ to be $x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and 0 for $x = 0$. Is f differentiable at $x = 0$? Prove your claim. If f is differentiable, is the derivative continuous?
- (5) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable. Prove that if f has a local maximum or minimum at c then $f'(c) = 0$.
- (6) Use the Intermediate Value Theorem, the Mean Value Theorem, and other facts, to prove that the function

$$f(x) = 4x^5 + x^3 + 2x + 1$$

has one and only one real zero.

- (7) Prove that the sum of the first n positive odd integers is equal to n^2 .
- (8) Evaluate $\lim_{n \rightarrow \infty} \sum_{i=1}^n (x_i)^2 \Delta x$, where $x_i = 1 + i/n$ and $\Delta x = 1/n$.
Do this directly without using integration.
- (9) Express $\lim_{n \rightarrow \infty} \sum_{i=1}^n (x_i^3 + x_i \sin(x_i)) \Delta x$, where $x_i = i\pi/n$ and $\Delta x = \pi/n$, as a definite integral, then evaluate it.
- (10) Prove that $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ when $|r| < 1$.
- (11) Find the intervals of convergence for the two series below.
a. $\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5}$ b. $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$
- (12) Evaluate $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$. Hint: Taylor series. Review Abel's Theorem about the end points of intervals of convergence.