A curve γ is piecewise C^1 if γ' exists and in continuous except for possibly finitely many points.

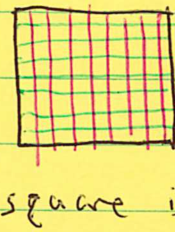
Thm (2.3.12) Deformation Thm Let $G \subset \mathbb{C}$ be open and let $f: G \to \mathbb{C}$ be differentiable.

(i) Let $z_0, z_1 \in G$. Suppose γ_0 and γ_1 are piecewise C^1 curves from z_0 to z_1 in G that are homotopic in G.

(ii) Suppose γ_0 and γ_1 are piecewise C^1 closed curves in G that are homotopic in G.

Then $$\int_{\gamma_0} f = \int_{\gamma_1} f.$$

Outline of Pf Since $[0,1]\times[0,1]$ is compact a cont. homotopy $H:[0,1]\times[0,1] \to G$ is uniformly cont. This allows us to find a partion of $[0,1]\times[0,1]$, fine enough, that



each smaller square is contained is a small open disk in G. In this disk we can apply Cauchy's Thm on a Disk.

Precing this together gives the result.

A corollary of this Theorem is

Thm (2.3.4) Let f be analytic on a region G. Let γ be a closed curve (piecewise C^1), which is homotopic to a point,

Then $\int_\gamma f = 0$.