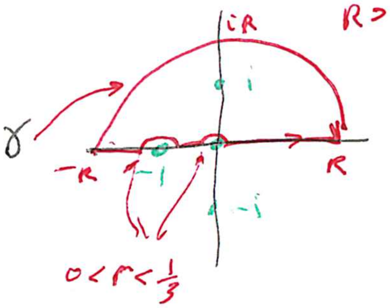
From 15' Ed,

Ch. 4 Review, #5 Compute $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x(x+1)(x^2+1)}\,dx$.

Solution "Complexify" to get $\displaystyle\int_\gamma \underbrace{\frac{e^{iz}}{z(z+1)(z^2+1)}}\,dz$, γ



$f(z) = \dfrac{g(z)}{h(z)}$

$h(z) = (z^2+z)(z^2+1) = z^4+z^3+z^2+z$

$h'(z) = 4z^3+3z^2+2z+1$

$h''(z) = 12z^2+6z$

$\mathrm{Res}(f,0) = \dfrac{g(0)}{h'(0)} = \dfrac{e^0}{1} = 1$ $\qquad \mathrm{Res}(f,-1) = \dfrac{g(-1)}{h'(-1)} = \dfrac{e^{-i}}{-4+3-2+1} = -\dfrac{e^{-i}}{2}$

$\mathrm{Res}(f,i) = \dfrac{g(i)}{h'(i)} = \dfrac{e^{-1}}{-4i-3+2i+1} = \dfrac{e^{-1}}{-2-2i} = \dfrac{1}{-2e}\,\dfrac{1}{1+i}\,\dfrac{1-i}{1-i} = \dfrac{-1}{4e}(1-i)$

$\displaystyle\int_\gamma f(z)\,dz = 2\pi i\left(\frac{-1}{4e}(1-i)\right) + \pi i\left(1-\frac{e^{-i}}{2}\right) = \frac{-\pi i}{2e} - \frac{\pi}{2e} + \pi i - \frac{\pi}{2}\underbrace{i e^{-i}}_{[i\cos(1)+\sin(1)]}$

$= \left[-\dfrac{\pi}{2e} - \dfrac{\pi}{2e}\sin(1)\right] + \left[\dfrac{-\pi}{2e} + \pi - \dfrac{\pi}{2}\cos(1)\right] i$

$= -\dfrac{\pi}{2e}\left[1+\sin(1)\right] + \dfrac{\pi}{2}\left[2 - \dfrac{1}{e} - \cos(1)\right] i$

Thus, $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x(x+1)(x^2+1)}\,dx = \frac{\pi}{2}\left[2-\frac{1}{e}-\cos(1)\right]$.