Summary of Section 1.4
Continuity

This is a long section that you should read carefully. I won't be able to cover all of it in class. Below is a summary of the major ideas.

I. Sets. Several types of sets in the complex plane are introduced.

(1) Open sets: $U \subset \mathbb{C}$ is open if for every $z \in U$ there is an $r > 0$ such that the disk of radius r and center z is in U. Unions of open sets are open. Finite intersections of open sets are open.
(2) Close sets: $C \subset \mathbb{C}$ is closed if $\mathbb{C} - C$ is open. Finite unions of closed sets are closed. Intersections of closed sets are closed. Note: $\mathbb{C}$ and $\emptyset$ are open and closed!
(3) Compact sets: $K \subset \mathbb{C}$ is compact if it is closed and bounded. There are equivalent definitions in terms of open covers and limit of sequences. If K is compact, C is closed and they are disjoint, then $\text{dist}(K, C) > 0$.
(4) Connected sets and Path Connected sets. PC $\implies$ C. Open & C $\implies$ PC.
A domian or a region refers to an open connected set (in this textbook).
(5) The Riemann sphere is $\mathbb{C} \cup \infty$. It is compact.

II. Functions and Limits.

(1) Limits: $\lim_{z \to c} f(z) = L$ means $\forall\, \epsilon > 0$, $\exists\, \delta > 0$ such that $0 < |c - z| < \delta \implies |f(z) - L| < \epsilon$.
(2) Continuity: $f : \mathbb{C} \to \mathbb{C}$ is continuous at c if $f(c) = \lim_{z \to c} f(z)$.
$f : \mathbb{C} \to \mathbb{C}$ is continuous a c if $\forall\, \epsilon > 0$, $\exists\, \delta > 0$ such that $|c - z| < \delta \implies |f(c) - f(z)| < \epsilon$.
(3) Uniformly continuous: $f : \mathbb{C} \to \mathbb{C}$ is uniformly continuous on $D \subset \mathbb{C}$ if $\forall\, \epsilon > 0$, $\exists\, \delta > 0$ such that $\forall c \in D$ we have $|c - z| < \delta \implies |f(c) - f(z)| < \epsilon$.
(4) Limits of sequences: $\lim_{n \to \infty} z_n = z$ means $\forall\, \epsilon > 0$, $\exists\, N$ such that $n > N \implies |z_n - z| < \epsilon$.
Limits are unique.
If f is continuous and $z_n \to z$, then $f(z_n) \to f(z)$.
Let C be closed; if $(z_n) \subset C$ and $z_n \to z$, then $z \in C$.

III. Functions and Sets.

(1) f is continuous $\Leftrightarrow$ inverse images of open sets are open $\Leftrightarrow$ inverse images of closed sets are closed.
(2) The continuous image of a compact set is compact.
(3) The continuous image of a connected set is connected.
(4) A function that is continuous on a compact set is uniformly continuous.
(5) A real valued continuous function on a compact set has a minimum and a maximum value.