

Section 1.3 Power Example

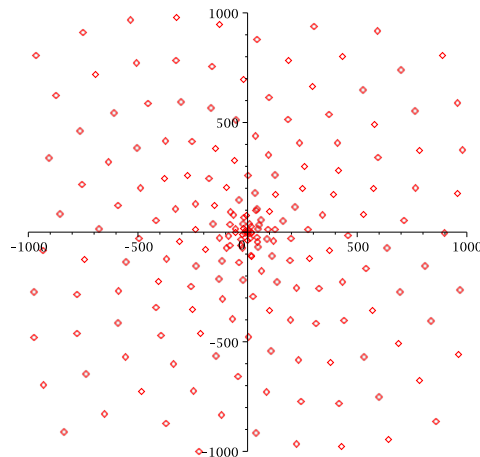
Problem: Describe the set $(1+i)^{\sqrt{2}+i}$.

Solution:

$$\begin{aligned}
 (1+i)^{\sqrt{2}+i} &= e^{(\sqrt{2}+i)\log(1+i)} \\
 &= e^{(\sqrt{2}+i)(\ln\sqrt{2}+\frac{\pi}{4}i+2\pi ni)} \\
 &= e^{\frac{\sqrt{2}\ln 2}{2} + \frac{\sqrt{2}\pi}{4}i + 2\sqrt{2}\pi ni + \frac{\ln 2}{2}i - \frac{\pi}{4} - 2\pi n} \\
 &= e^{\frac{\sqrt{2}\ln 2}{2} - \frac{\pi}{4} - 2\pi n} e^{i(\frac{\sqrt{2}\pi}{4} + \frac{\ln 2}{2} + 2\sqrt{2}\pi n)}
 \end{aligned}$$

Let $r_n = e^{\frac{\sqrt{2}\ln 2}{2} - \frac{\pi}{4} - 2\pi n}$ and $\theta_n = \frac{\sqrt{2}\pi}{4} + \frac{\ln 2}{2} + 2\sqrt{2}\pi n$.

For example, $r_0 \approx 0.744$ and $\theta_0 \approx 1.457$ radians or 83.5 degrees. We can plot this in polar coordinates. For other values of n the exponential growth of r_n makes it very difficult to see more than a handful of points on any plot. So, instead I plotted $(\ln r_n, \theta_n)$ and got the plot below.



I used Maple. Here are the commands.

```

> points:= seq([ln(2)/sqrt(2)-Pi/4-2*Pi*n,Pi/2/sqrt(2)+ln(2)/2+2*sqrt(2)*Pi*n],n=-200..20):
> pointplot(points,coords=polar,color=red,coordinateview=[-1000..1000,-1000..1000]);

```

Other plotting programs would have similar commands.