We explore the map T(z) = z + 1/z.

```
> with(plots):with(plottools):
> T:= z -> z+1/z;
```

$$T := z \to z + \frac{1}{z} \quad (1)$$

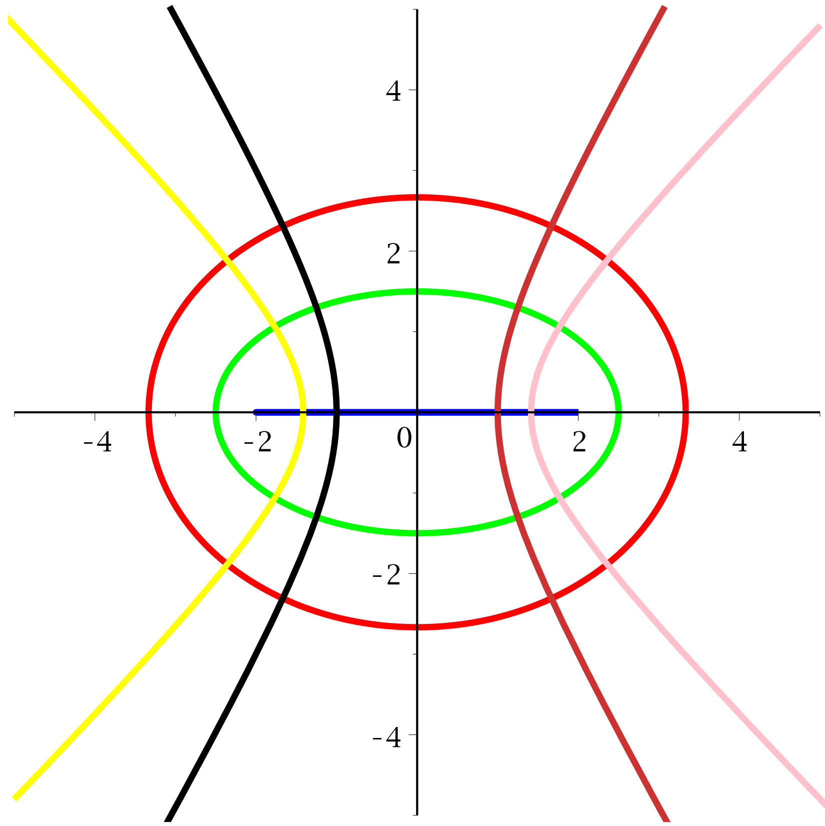
Below we plot the images from T of circle and rays.
C1, is the image of a circle of radius 1 centered at the origin. It is just the blue line seqment.
C2 is the image of a circle of radius 2 centered at the origin.. It is the green ellipse.

C3 is the image of a circle of radius 3 centered at the origin.. It is the red ellipse.
R1 is the image of a ray based at the origin with angle Pi/4. It is the pink hyperbola.
R2 is the image of a ray based at the origin with angle 5Pi/4. It is the yellow hyperbola.
R3 is the image of a ray based at the origin with angle Pi/3. It is the orange hyperbola, although it looks brown.
R4 is the image of a ray based at the origin with angle 7Pi/6. It is the black hyperbola.

Here is a Youtube video where it is proven T take circles centered at the origin to ellipses and lines through the origin to hyperbolas.

https://www.youtube.com/watch?v=4bhdMM66u-E

```
> C1:=complexplot(T(1*cos(t)+I*1*sin(t)),t=0..2*Pi,thickness=3,
  color=blue):
> C2:=complexplot(T(2*cos(t)+I*2*sin(t)),t=0..2*Pi,thickness=3,
  color=green):
> C3:=complexplot(T(3*cos(t)+I*3*sin(t)),t=0..2*Pi,thickness=3,
  color=red):
> R1:=complexplot(T(r*cos(Pi/4)+I*r*sin(Pi/4)),r=0..10,thickness=3,
  color=pink):
> R2:=complexplot(T(r*cos(Pi+Pi/4)+I*r*sin(Pi+Pi/4)),r=0..10,
  thickness=3,color=yellow):
> R3:=complexplot(T(r*cos(Pi/3)+I*r*sin(Pi/3)),r=0..10,thickness=3,
  color=orange):
> R4:=complexplot(T(r*cos(Pi/6+Pi/2)+I*r*sin(Pi/6+Pi/2)),r=0..10,
  thickness=3,color=black):
> display(C1,C2,C3,R1,R2,R3,R4,view=[-5..5,-5..5]) ;
```



Now we look at an inverse. The textbook's formual for the inverse is wrong in two ways. The formal in table 5.2.10 for the inverse is (z+sqrt(z^2-1))/2.
First, the 1 should be a 4. Second, the plus should be + or -, and which you use is tricky. Below I use the sign function to get the coreect sign. We will discuss this a bit in class.

First, we look at the inverse image of the real line. It is graphed in black below.

```
> Tinv0:= x -> (x+'sign(x+2)'*sqrt(x^2-4))/2;
```

$$Tinv0 := x \rightarrow \frac{1}{2}\, x + \frac{1}{2}\, 'sign(x+2)'\, \sqrt{x^2-4} \quad (2)$$

```
> p0:=complexplot(Tinv0(x),x=-3..3,view=[-3..3,-1..2],numpoints=
  1000,color=black);
```

$$p0 := PLOT(\ldots) \quad (3)$$

Then we feed in three lines, z=x+I/10, z=x+I/2 and z = x+I. Their inverse images are plotted in brown below.
In Section 5.3 we apply this to a fuild flowing past a barrior.

```
> TinvI:= x -> (x+'sign(x)'*sqrt(x^2-4))/2;
```

$$TinvI := x \to \frac{1}{2}\,x + \frac{1}{2}\,\text{'}sign(x)\text{'}\sqrt{x^2-4} \qquad \textbf{(4)}$$

```
> p1:=complexplot(TinvI(x+0.1*I),x=-3..3,view=[-3..3,-1..2]);
```

$$p1 := PLOT(\ldots) \qquad \textbf{(5)}$$

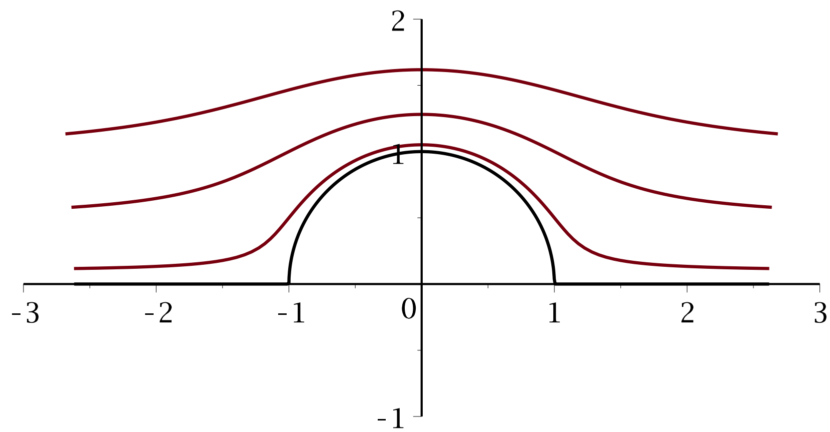
```
> p2:=complexplot(TinvI(x+0.5*I),x=-3..3,view=[-3..3,-1..2]);
```

$$p2 := PLOT(\ldots) \qquad \textbf{(6)}$$

```
> p3:=complexplot(TinvI(x+1.0*I),x=-3..3,view=[-3..3,-1..2]);
```

$$p3 := PLOT(\ldots) \qquad \textbf{(7)}$$

```
> display(p0,p1,p2,p3);
```




```
>
```